Examination Performance Analysis (et4-367)

Question 1: A router supports two QoS classes and for each class packets arrive according to a Poisson process with rate λ_i for $i = 1, 2$. These two Poisson processes N_i for $i = 1, 2$ are independent and the Poisson process rates are $\frac{1}{6}$ and $\frac{1}{3}$ packets per minutes, respectively. Suppose that the router is switched off for a period of 30 minutes.

(a) What is the probability density function of the total number of packets of both classes that will arrive during the switched-off period?

(b) Given that at a router exactly one arrival has occurred during the switched-off interval, what is probability that that arrival came from class 1 whose arrival rate is λ_1?

Question 2: A faulty digital video conferencing system shows a clustered error pattern. If a bit is received correctly, then the chance to receive the next bit correctly is 0.999. If a bit is received incorrectly, then the next bit is incorrect with probability 0.95.

(a) Model the error pattern of this system using the discrete-time Markov chain.

(b) Is this Markov chain irreducible?

(c)In the long run, what is the fraction of correctly received bits and the fraction of incorrectly received bits?

(d)After the system is repaired, it works properly for 99.9% of the time. A test sequence after repair shows that, when always starting with a correctly received bit, the next 10 bits are correctly received with probability 0.9999. What is the probability now that a correctly (and analogously incorrectly) received bit is followed by another correct (incorrect) bit?

Question 3: A small company uses a telephony system with a switch that can handle 2 simultaneous phone calls. The company employs 60 people and the arrival of call requests at the switch can be closely modeled according to a Poisson process. On average, each employee makes 2 phone calls per 8 hours. We assume that the call duration is exponentially distributed with mean 4 minutes. When all the lines are busy, the switch queues the call requests until a line becomes available. Round the answers to the problems to two digits.

(a) Describe the system with the Kendal notation. What is the mean arrival rate λ and what is the mean service rate μ per phonecall in unit of calls/minute? What is the load ρ?

(b) Describe the system as a birth and death process, where each state refers to the number of calls. Draw the state transition diagram. Compute π_0 the probability that no packet is in the system in the steady state (the intermediate steps to compute π_0 are required).

(c) What is the probability that an employee needs to wait for a line?

(d) The telephony system is updated such that calls that cannot be serviced by the switch are redirected through public telephone lines. What proportion of calls is redirected?

Question 4: An ISP wants to have an idea of the end-to-end delay D along a path in the Internet. Assume the delay T incurred in each link is an i.i.d. exponential random variable with mean $1/\alpha$. The random variable $D = \sum_{j=1}^{H_N} T_j$, where T_j is the delay in each constituent link of the path and H_N is the hopcount.

(a) Consider a specific path with a constant hopcount $H_N = k$. What is the generating function of the end-to-end delay $\sum_{j=1}^{k} T_j$?

(b) The ISP learned from Performance Analysis that the hopcount H_N of an arbitrary path in the Internet is not a constant but approximately follows the generating function

$$\varphi_{H_N}(z) = \frac{N}{N-1}\left(\frac{\Gamma(N+z)}{N!\Gamma(z+1)} - \frac{1}{N}\right)$$

Assume that the hopcount of a path is independent of the delay incurred in a link. Compute the generating function of the end-to-end delay $D = \sum_{j=1}^{H_N} T_j$ in this case. *Hint*: Use the law of total probability on the pgf of D.