Parallel Algorithms and Parallel Computers

(in4026)

Examination April 13, 2010

14.00-16.00 hrs

***IMPORTANT**: Please use a separate sheet for each question!*

PART I

Question 1 *(25 points)*

(5 points)

a. Consider a complete Tree network (i.e. all the nodes count). Derive expressions for the diameter, the bisection width, the arc connectivity, and the number of links.

(5 points)

b. What is a "perfect shuffle" interconnection pattern? Draw a perfect shuffle interconnection pattern with 8 inputs and 8 outputs.

(10 points)

c. Let the cost of a network be proportional to the total number of wires. Given are two p-processor networks: a p-processor ring and a p-processor hypercube. The two networks have equal cost (i.e. the same total number of wires per network). Derive the average communication latencies of sending a message of size m using cut-through routing for both networks. (Hint: one of the two networks may need multiple wires per link to obtain equal cost for both networks and more wires per link give proportional more communication bandwidth per link).

(5 points)

d. Explain the working of the E-cube routing algorithm in Hypercubes. Show two ways how in a Hypercube with dimension *4* ($d=4$) a message is routed from node *0010* to node *1101* by using this algorithm. Do both routes have the same length?

Question 2 *(25 points)*

(5 points)

a. Given is a $p = 2^d$ processor Hypercube. What is the communication time of a message of length m for an *all-to-all* broadcast using *cut-through* routing.

(5 points)

b. Consider a reduction operation on a Hypercube with p processors, in which each processor contains a data item to be reduced. What is the communication time of the reduction operation (you may use either *store-and-forward* or *cut-through* routing)? The data items subject to reduction have length m.

(5 points)

c. Given is that the cost of a parallel algorithm and its workload are $C = O(n^2)$ and $W = n\log(n)$, respectively. Is this parallel algorithm Cost Optimal? Explain your answer.

(10 points)

d. Given is the addition of n numbers on a p processor Hypercube (where p divides n). Given is also that one addition takes 1 unit of time and a single communication step to another processor takes 5 units of time.
   - Derive the overhead function T_o
   - Derive the number of processors for which the execution time of the addition algorithm in the previous exercise is minimal.
   - Under which relation between p and n is the minimal execution time cost-optimal?

End of Examination