TECHNISCHE UNIVERSITEIT DELFT
Faculteit Elektrotechniek, Wiskunde en Informatica

Exam IN4301 Advanced Algorithms – Part I of III

October 1, 2015, 9:00–10:00

- This is a closed book examination with 4 questions worth of 20 points in total.
- Your mark for this exam part will be the number of points divided by 2.
- If your average for all three exam parts is at least a 5.0, the average of the programming exercises, as well as the average of the homework exercises, your final mark for this course is the average of these three marks, rounded to the nearest half of a whole number. That is, 9.7 is rounded to 9.5, and 5.8 is rounded to 6.
- Use of book, readers, notes, and slides is not allowed.
- Use of (graphical) calculators is not permitted.
- Specify your name, student number and degree program, and indicate the total number of submitted pages on the first page.
- Write clearly, use correct English, and avoid verbose explanations. Giving irrelevant information may lead to a reduction in your score.
  Notice that almost all questions can be answered in a few lines!
- This exam covers Chapters 10 of Kleinberg, J. and Tardos, E. (2005), *Algorithm Design*, all information on the slides of the course of the first 4 lectures, and the set of papers as described in the study guide.
- The total number of pages of this exam is 1 (excluding this front page).

1. (a) (1 point) Describe the general idea of a search tree algorithm (in about 3 lines).
   (b) (2 points) Explain how an upper bound on the run time of a search tree algorithm can be obtained (in about 6 lines).

2. Consider the problem of deciding whether there exists a VERTEX COVER of size at most k in a given undirected graph G.
   (a) (3 points) Describe (with three simple rules) how to reduce the input to an instance (G', k') such that
      1. $k' \leq k$
      2. $|G|$ is smaller than k'^2
      3. (G', k') has a solution if and only if (G, k) has one, and
      4. the reduction from (G, k) to (G', k') must be computable in polynomial time.
   (b) (3 points) Prove that if a vertex cover exists, your reduced instance (G', k') has at most k'^2 vertices.

3. Consider the problem of computing the length of the shortest tour through all cities $\{1, 2, \ldots, n\}$ given pairwise distances d_{ij} for $i, j \in \{1, 2, \ldots, n\}$.
   (a) (3 points) Give a recursive formulation of the length of the shortest path from city 1 to a city i. (Make sure to include the base case.)
   (b) (1 point) Express the length of the shortest tour through all cities using the function you defined in the previous question.
   (c) (1 point) Suppose your recursive function is implemented using dynamic programming. Give a tight upper bound on the (memory) space required. Briefly explain your answer.

4. Let $(Tr = (T, F), \{V_t : t \in T\})$ be a clique tree of $G = (V, E)$. In this question you are asked to show (in three steps) that Tr has width equal to the tree width $tw(G)$.
   (a) (1 point) First give the definition of a *clique tree* of a graph G (you may use other terms defined in the course).
   (b) (3 points) Show that if G contains a clique S of vertices, $tw(G) \geq |S| - 1$.
   (c) (2 points) Show now that Tr has width equal to $tw(G)$.

End of test