IN4301 Advanced Algorithms Exam – Part II

January 26, 2015, 14:00–16:00

- Please answer questions 1 and 2 on a **separate sheet** from questions 3 and 4.
- This is a closed book examination with 4 questions worth 50 points in total. It covers the material in the **second half of the course**.
- Use of books, readers, notes, slides and (graphical) calculators is not allowed.
- Your grade for this part of the exam will be the number of points awarded, divided by 5.
- Write your name, student number, degree program, and number of submitted sheets of paper on the first page.
- Write clearly, in correct English, and avoid verbose explanations. Giving irrelevant information may lead to a reduction in your score.
- The total number of pages of this exam is 3.

Questions

1. Let $G = (V, E)$ be a directed graph with vertex set V and edge set E, and let $in(v)$ be the set of incoming edges of vertex v and $out(v)$ the set of outgoing edges of v. Each edge $e \in E$ has capacity c_e. One way of formulating the max flow problem from s to t in $G = (V, A)$ as an optimization problem is as follows:

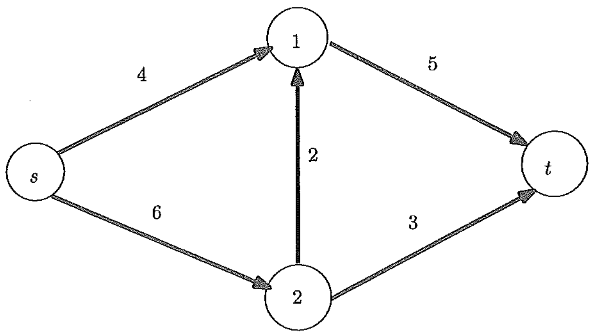
$$
\begin{array}{rcl}
 & \max f & \\
\text{subject to} & \sum_{e \in out(s)} x_e - f & = & 0 \\
 & -\sum_{e \in in(t)} x_e + f & = & 0 \\
 & \sum_{e \in out(v)} x_e - \sum_{e \in in(v)} x_e & = & 0 \text{ for all } v \in V \setminus \{s,t\} \\
 & 0 \le x_e & \le & c_e \text{ for all } e \in E
\end{array}
$$

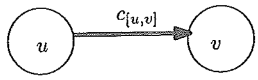
Consider the specific instance of the max flow problem based on the following graph: (NB: The number on an edge is the edge's capacity.) The formulation for this specific problem is:

$$
\begin{array}{lccccccccccccc}
\max & & & & & & & & & & & f & & \\
\text{subject to} & x_{[s,1]} & + & x_{[s,2]} & & & & & & & - & f & = & 0 \\
 & & & & & & - & x_{[1,t]} & - & x_{[2,t]} & + & f & = & 0 \\
 & -x_{[s,1]} & & & - & x_{[2,1]} & + & x_{[1,t]} & & & & & = & 0 \\
 & & - & x_{[s,2]} & + & x_{[2,1]} & & & + & x_{[2,t]} & & & = & 0 \\
 & x_{[s,1]} & & & & & & & & & & & \le & 4 \\
 & & & x_{[s,2]} & & & & & & & & & \le & 6 \\
 & & & & & x_{[2,1]} & & & & & & & \le & 2 \\
 & & & & & & & x_{[1,t]} & & & & & \le & 5 \\
 & & & & & & & & & x_{[2,t]} & & & \le & 3
\end{array}
$$

$$x_{[u,v]} \ge 0 \text{ for all } [u,v] \in E$$

(a) (5 points) Formulate the dual of the specific problem instance given above.





(b) (5 points) Formulate the dual of the general max flow formulation.

(c) (1 point) Which network problem is modeled by the dual of the max flow problem?

(d) (4 points) Indicate for each of the following statements whether they are true or false. (No motivation needed).

(i) Given is a class P of integer minimization problem. We have a specific instance I of P and have computed the ratio between the integer optimal value, $z_{IP}(I)$, and the value of the LP-relaxation for I, $z_{LP}(I)$. This ratio is equal to 2. The integrality gap for P is therefore greater than or equal to 2.

(ii) If an integer optimization problem (minimization) has integrality gap equal to ρ, then it is possible to design an approximation algorithm with performance guarantee strictly less than ρ purely based on the linear relaxation.

(iii) If a primal problem is infeasible, then the dual is *always* infeasible as well.

(iv) Given are feasible solutions x and y to a primal-dual pair of problems. If the solutions satisfy the complementary slackness conditions, then we can conclude that x and y are optimal for their respective problem.

2. (10 points) Given an undirected graph $G = (V, E)$ with nonnegative weights w_{ij} on the edges $\{i,j\} \in E$, the maximum cut problem is the problem of determining a cut (a partition of the vertices $(W, V \setminus W)$) in G such that the sum of the weights of the edges $\{i,j\}$, such that $i \in W$ and $j \in V \setminus W$ is maximized. This problem can be formulated as a quadratic integer optimization problem (QIP). Goemans and Williamson formulated a relaxation (RP) of this QIP:

$$(RP): \quad \max \frac{1}{2} \sum_{i<j}^{n} w_{ij}(1 - v_i v_j)$$
$$\text{s.t. } v_i \in R^n, \; \|v_i\| = 1, \qquad i \in V.$$

Formulate a semidefinite optimization problem that is quivalent to (RP). Explain why the semidefinite problem is equivalent to (RP) and how the solution to (RP) can be retrieved from the solution to the semidefinite problem.

Please answer the following questions on a separate sheet.

3. (a) In her Feature Article "Toward an Experimental Method for Algorithm Simulation," Catherine McGeoch has the following figure, which describes the *general* paradigm of simulation research.

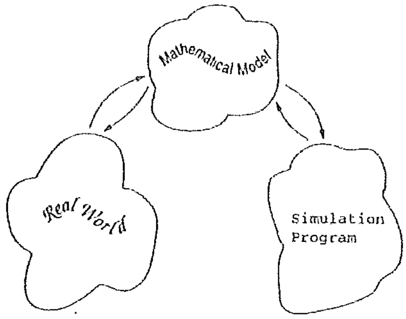


Figure 1: The Paradigm of Simulation Research.

McGeoch applies this paradigm to the *study of algorithms*. According to McGeoch,

i. (3 points) what do the three blobs in the figure then refer to?

ii. (4 points) based on this figure, how does Simulation Research on algorithms work?

(b) (6 points) McGeoch writes that "Computational experiments can be developed to target various kinds of questions about performance. The bin packing study illustrates three modes of experimentation." Which three modes (or: types of experimental study) does she mention? Describe each in one sentence.

4. (a) (2 points) In his paper "Needed: An Empirical Science of Algorithms," Hooker sees two "steps in the right direction" that researchers in the OR and computer science communities have taken toward an empirical science of algorithms. The second step is "the *heuristic* use of experimentation". What is the first step he mentions?

(b) (3 points) What does Hooker mean by "heuristic use of experimentation"?

(c) (3 points) Give an example of how McGeoch makes "heuristic use of experimentation" in her study of the FFD algorithm for Bin Packing.

(d) (4 points) Hooker writes that "*experimental* mathematics is not necessarily *empirical* mathematics." What, according to Hooker, characterizes an *empirical* discipline?

End of exam questions