IN4301: Test Exam

October 21, 2008

Question 1 (Set Basis)

Given a finite collection C of finite sets, and an integer k. Decide whether there exist a finite collection B of k sets such that every set $A \in C$ equals the union of some subset of B.

1. Given $C = \{\{1,2,3\},\{1,3,4\},\{2,3\},\{2,3,4\}\}$ and $k = 3$. Give a set B that meets the criteria given above.
2. If the answer to a set-basis instance is “yes”, what can you say about the number of sets in C in relation to k?
3. If two elements x and y appear in precisely the same family of sets in C (i.e., for all $S \in C$, $x \in S$ if and only if $y \in S$), show that removing y from all sets in C preserves the answer to set basis (i.e. an instance in the problem with y is a yes-instance if and only if the problem without y is a yes-instance).
4. Show that set basis is fixed-parameter tractable by reducing to a problem kernel of size $f(k)$.

Question 2 (Tree decomposition of triangulated cycle graphs)

(See Ex.10.4 from Kleinberg)

Consider the problem of finding a tree decomposition of a triangulated cycle graph $G = (V, E)$. (See the exercise in the book for a definition.) Show that the tree width of such graphs is 2, and give an efficient algorithm to construct such a tree decomposition.

Question 3 (Approximation algorithms)

Analyse the performance of the following approximation algorithm A for MINBINPACKING:

Iteratively, put the item in the first bin into which it fits.

1. Show that for every instance x, we have $c(A(x)) \leq 2.opt(x) + 1$;

2. Show that $5/3$ is a lower bound on the performance of A.

Question 4 (Matroids)

1. Let $G = (V, E)$ be an undirected graph. Let (S, I) be a subset system where $S = E$ and $A \subseteq E$ is independent iff the subgraph $G(A)$ generated by A is acyclic.

   (a) What are the bases in this graphic matroid and how can you characterize dependent sets?

   (b) Given a weight function $w : E \to N$, give a greedy algorithm to select an optimal basis for the graphic matroid.

   (c) What is the time-complexity of this greedy algorithm?

2. Let $M = (S, I)$ be a matroid. Prove or disprove the following statements:

   (a) if $T \subseteq S$ then $M' = (T, I \cap 2^T)$ is a matroid.

   (b) if $J \subseteq I$ then (S, J) is a matroid.

   (c) if k is an arbitrary number and $I' = (X \in I||X| \leq k)$ then (S, I) is a matroid.

   (d) if $M' = (S', I')$ is a matroid then $M \cup M' = (S \cup S', I \cup I')$ is a matroid.