DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics, and Computer Science
Information and Communication Theory Group

EXAMINATION MULTIMEDIA COMPRESSION

Tuesday July 1, 2008 (9 am –12 noon)

The questions are posed in English. Answers to open questions (Section B) can be either in Dutch or English, depending on your preference. Keep your answers as concise as possible. Answers to the multiple choice questions (Section A) should be written on a separate sheet of paper.

This exam has an open questions and a multiple choice section

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A. Multiple Choice Questions Section (30 questions)

Evaluate the statements given, or find the question to the posed question(s), then select the correct answer (a, b, c, or d) and fill out the answer on the answer form. Do not forget to hand-in this answer form.

GENERAL COMPRESSION

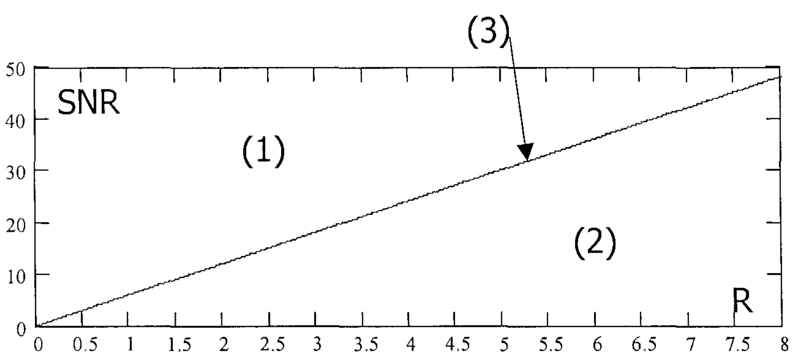
A1. Statement I : Uncorrelated signals have to be compressed lossy.
Statement II : The compression factor for correlated signals is always larger than for uncorrelated signals.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A2. We consider the compression of a memoryless Gaussian distributed signal X. It is known that the relation between bit rate and distortion is described by the following rate-distortion function:

$$R = \frac{1}{2}\log_2\left(\frac{\sigma_x^2}{\sigma_q^2}\right).$$

Using this function we create the following bit rate versus SNR graph:



Statements:

(I) Any scalar quantizer has a performance in area (2). A scalar quantizer exists that has a performance arbitrarily close to the line indicated by (3).
(II) Any compression system has a performance that lies in area (2). A compression system exists that has a performance arbitrarily close to the line indicated by (3).
(III) Any scalar quantizer has a performance in area (1). A scalar quantizer exists that has a

performance arbitrarily close to the line indicated by (3).

(IV) Any compression system has a performance that lies in area (1). A compression system exists that has a performance arbitrarily close to the line indicated by (3).

Answers:
(a) Only II is correct
(b) Only III and IV are correct
(c) Only I and III are correct
(d) Only III is correct

A3. The picture below shows an compressed (and decompressed) image that has been received with bit errors in the compressed stream.

The compression technique used is

Answers:
(a) DPCM
(b) Vector quantization
(c) Transform coding
(d) Subband coding

LOSSLESS COMPRESSION

A4. We consider the Lempel-Ziv (LZ78) coding of the following bit string:

0 0 1 0 1 1 0 1 1 1 0 1 0 1 0 0 1 ...

The LZ78 parsing of this bit string is:

Answers:
(a) 0 | 0 | 1| 0 | 1 1 | 0 | 1 1 1 | 0 | 1 0 | 1 0 | 0 | 1 ...
(b) 0 | 0 1 | 0 1 1 | 0 1 1 1 | 0 1 0 | 1 | 0 0 | 1 ...
(c) 0 0 | 1 0 | 1 1 0 | 1 1 1 0 | 1 0 | 1 0 | 0 | 1 ...
(d) 0 | 0 1 | 0 1 | 1 | 0 1 | 1 1 | 0 1 | 0 1 | 0 | 0 1 ...

A5. Given are the four VLC codes for a 5-level scalar quantizer designed for a signal with $\mu = 0$ and $\sigma = 1$:

Number	**Representation Level**	**Probability**	**VLC code 1**	**VLC code 2**	**VLC code 3**	**VLC code 4**
1	-2.25	0.055	00	0010	0010	001
2	-0.84	0.221	10	01	000	01
3	0.00	0.448	0	1	1	1
4	0.84	0.221	01	001	01	0001
5	2.25	0.055	11	0000	0011	0000

The uniquely decodable VLC code that yields the shortest average codeword length is:

Answers:
(a) VLC code 1
(b) VLC code 2
(c) VLC code 3
(d) VLC code 4

A6. Statement I : Variable length encoding is a lossless coding technique.
Statement II : Quantization is a lossless coding technique.

Answers: (a) I and II are both correct (c) Only II is correct
(b) Only I is correct (d) Neither I nor II is correct

QUANTIZATION

A7. Given is the 5-level scalar Lloyd-Max quantizer designed for a signal with $\mu = 0$ and $\sigma = 1$:

Number	Representation Level	Probability
1	-2.25	0.055
2	-0.84	0.221
3	0.00	0.448
4	0.84	0.221
5	2.25	0.055

Statement I: The quantizer is a midrise quantizer.
Statement II: The quantizer is has a decision level at 0.42.

Answers: (a) I and II are both correct (c) Only II is correct
(b) Only I is correct (d) Neither I nor II is correct

A8. Statement I : The entropy of a scalar quantizer's representation levels is always smaller than or equal to the average codeword length of a Huffman code.
Statement II : The average codeword length of a Huffman code of a symmetric scalar quantizer with 3 or more representation levels is always larger than 1 bit per symbol.

Answers: (a) I and II are both correct (c) Only II is correct
(b) Only I is correct (d) Neither I nor II is correct

A9. Given is the following scalar quantizer:

Number	Representation Level	Probability
1	-2.03	0.054
2	-1.18	0.137
3	-0.56	0.199
4	0.00	0.220
5	0.56	0.199
6	1.18	0.137
7	2.03	0.054

The entropy $H(X)$ of this quantizer is equal to:

Answers: (a) 1.836 bit/quantizer level (c) 2.808 bit/ quantizer level
(b) 2.648 bit/quantizer level (d) 3.000 bit/ quantizer level

A10. We apply vector quantization to a 1-D signal $x(k)$. The code book is given by the following four code book vectors:
$c_1 = (3, 2)$ $c_2 = (-1, -1)$ $c_3 = (0, 2)$ $c_4 = (0, -3)$

The average bit rate at which $x(k)$ will be compressed is equal to

Answers: (a) 2 bit / sample (b) 1 bit / sample (c) 0.5 bit / sample (d) Not enough information to answer this question

A11. In vector quantization of images, blocks of size 8x8 are used. The overall bit rate is 0.5 bit per sample.

Statement I : The bit rate per vector is 0.0078 bit per vector

Statement II : The code book contains more than 1 million code book vectors.

Answers: (a) I and II are both correct (b) Only I is correct (c) Only II is correct (d) Neither I nor II is correct

A12. In vector quantization, a code book consists of the following vectors. Per vector, the probability of selecting that particular code book vector during the VQ of the image 'Lena' is given.

Code Book Vector Number	**Code vector**	**Probability**
1	(101, 120, 89, 25)	0.10
2	(10, 100, 205, 209)	0.50
3	(240, 13, 78, 178)	0.30
4	(120, 100, 67, 30)	0.10

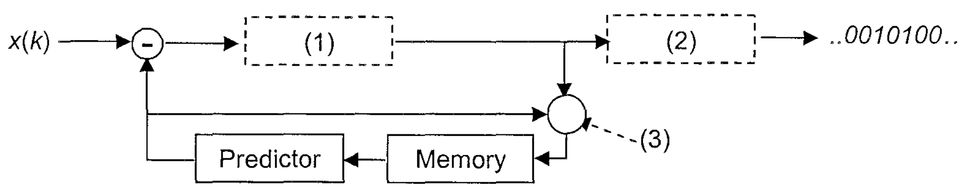
Statement I : The VQ bit rate when encoding the code book numbers using a fixed length code is 2 bit/sample

Statement II : The VQ bit rate when encoding the code book numbers using a VLC code is at least 0.4 bit/sample

Answers: (a) I and II are both correct (b) Only I is correct (c) Only II is correct (d) Neither I nor II is correct

DPCM

A13. Given is the following DPCM system



Answers:

(a) The quantizer should be at position (1), and the operation at (3) should be addition (+)
(b) The quantizer should be at position (2), and the operation at (3) should be addition (+)
(c) The quantizer should be at position (1), and the operation at (3) should be subtraction (-)
(d) The quantizer should be at position (2), and the operation at (3) should be subtraction (-)

A14. Given are the following two-dimensional predictors (the index i indicates the line number, the index j the column number in the picture):

Predictor 1: $x(i,j) = 0.2x(i,j-1) - 0.2x(i-1,j-1) + 0.2x(i-1,j)$

Predictor 2: $x(i,j) = 0.2x(i-1,j) + 0.8x(i,j)$

Statement I : Predictor 1 can be used for DPCM encoding of images.
Statement II : Predictor 2 can be used for DPCM encoding of images.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A15. The first 4 values of the signal $x(k)$ are given as: $x(1)=2$, $x(2)=1$, $x(3)=1$, $x(4)=0$. The signal $x(k)$ is DPCM encoded. The DPCM encoder uses a 1 bit quantizer with representation levels ± 1, and a first order linear predictor with prediction coefficient $h_1=1/2$. The value $x(0)$ is assumed to be equal to $x(0)=0$.

Statement I : The first 4 predicted signal values $\hat{x}(k)$ are equal to 0.0, 0.5, 0.75, 0.87
Statement II : The first 4 reconstructed signal values $\tilde{x}(k)$ are equal to 1.0, 1.5, 0.75, -0.25

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A16. We consider the DPCM coding of a Gaussian distributed signal $x(k)$ with the autocorrelation function $R_X(k)$. The variance of $x(k)$ is $\sigma_x^2 = 1600$, and the variance of the prediction difference $\Delta x(k)$ is $\sigma_{\Delta x}^2 = 400$.

Statement I: The theoretically expected gain of DPCM over PCM is 2 bit/sample.
Statement II: The larger the variance of a signal is, the larger the prediction gain will be.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A17. Given is a signal $x(k)$ with the following autocorrelation function:

$$R_X(k) = 12\,(0.7)^{|k|}$$

The signal $x(k)$ is encoded using DPCM. The DPCM system makes use of a first order linear predictor with prediction coefficient h_1.

Statement I: The optimal predictior coefficient is $h_1=0.3$.
Statement II: The prediction gain is equal to 100/51.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A18. We consider an image for which the following values of the 2-dimensional autocorrelation function $R_x(i,j)$ have been calculated

$$R_X(i,j) = \begin{bmatrix} R_X(-1,-1) & R_X(-1,0) & R_X(-1,1) \\ R_X(0,-1) & R_X(0,0) & R_X(0,1) \\ R_X(1,-1) & R_X(1,0) & R_X(1,1) \end{bmatrix} = \begin{bmatrix} 8.27 & 8.7 & 8.27 \\ 9.5 & 10.0 & 9.5 \\ 8.27 & 8.7 & 8.27 \end{bmatrix}$$

If we use the following 2-D predictor in a DPCM system:

$$x(i,j) = \alpha x(i,j-1) + \beta x(i-1,j)$$

then the optimal prediction coefficients α and β are equal to:

	Answers:	(a) $\alpha = 0.155$ and $\beta = 0.822$ (b) $\alpha = 0.822$ and $\beta = 0.155$ (c) $\alpha = 0.267$ and $\beta = 0.729$ (d) $\alpha = 0.729$ and $\beta = 0.267$

TRANSFORM CODING

A19. Statement I: The efficiency of DCT-based image compression is independent of the block size (e.g. 4x4, 8x8).

Statement II: The DCT is the optimal correlation-reducing transform.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

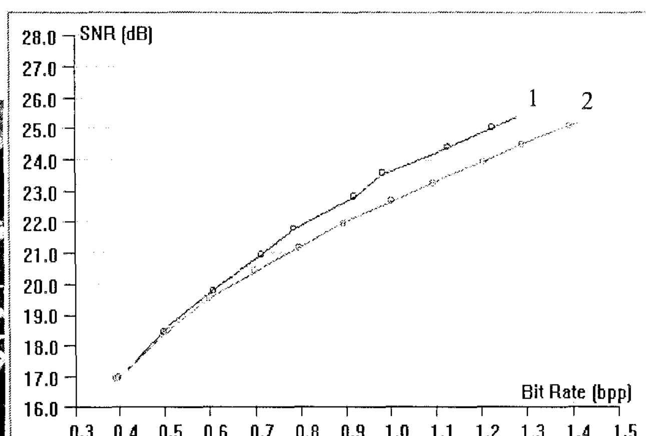
A20. Given is the following decorrelating transform T for the case N=4:

$$T^t = b\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & a & -1 \end{bmatrix}$$

Answers: To make the transform T orthogonal, we need

(a) $a = 1$ and $b = 0.5$
(b) $a = 1$ and $b = 0.25$
(c) $a = -1$ and $b = 0.5$
(d) $a = -1$ and $b = 0.25$

A21. The following figure shows the bit rate-SNR performance for the JPEG compression of the 'Building' image using a different normalization (quantization or weighting) matrix. In one case the standard JPEG normalization matrix has been used, in the other case no normalization matrix (or similarly, a normalization matrix filled with identical values) has been used.



Statement I : Curve "1" is the result of JPEG using the standard JPEG normalization matrix.

Statement II : The answer to the question "which of the two normalization matrices yields the best bit rate-SNR performance" is image dependent.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

22.	The quantization error σ_q^2 of any 4x4 image block $x(i,j)$ that is subject to a 4x4 DCT transform can be written as follows: $$\sigma_q^2 = \sum_{i=1}^{4}\sum_{j=1}^{4}\left[\ldots - \sum_{k=1}^{4}\sum_{l=1}^{4}\hat{\theta}_{kl}W_{kl}(i,j)\right]^2$$ where $\hat{\theta}_{kl} = Q(\theta_{kl})$ are quantized DCT coefficients and $W_{kl}(i,j)$ are the DCT basis-images. At the position of the dots (...) in the above equation, we need to fill in: Answers: (a) θ_{kl} (b) $x(i,j)$ (c) $\theta_{kl}W_{kl}(i,j)$ (d) The entire equation is incorrect
A23.	In JPEG and in MPEG, the quantization of DCT coefficients is carried out using uniform quantizers with coarseness determined by the quantizer step size Δ. Furthermore a weight (normalization or quantization) matrix is used. Statement I : Higher frequency DCT coefficients are usually quantized with a coarser quantizer. Statement II : If the entries in the weight matrix N_{kl} are given by $$N_{kl} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$ then the quantization of the DCT coefficients can be written as $$\hat{\theta}_{kl} = Q[\theta_{kl}] = \text{round}\left(\frac{\theta_{kl}}{\Delta} N_{kl}\right)$$ Answers: (a) I and II are both correct (b) Only I is correct (c) Only II is correct (d) Neither I nor II is correct
A24.	Given is a signal $x(k)$ with zero-mean, of which the following autocorrelation coefficients are given: $R_x(0) = 5$ and $R_x(1) = 3$. Consider the following decorrelating transform: $$\mathbf{T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$ The transform is applied to the signal $x(k)$, yielding the (sets of) decorrelated transform coefficients θ_1 and θ_2. We wish to encode the signal $x(k)$ at an average bit rate of 2 bit per sample. The optimal bit allocation result for θ_1 and θ_2, denoted as $(R_{\theta_1}, R_{\theta_2})$, is Answers: (a) $(R_{\theta_1}, R_{\theta_2}) = (4, 0)$ (b) $(R_{\theta_1}, R_{\theta_2}) = (2.5, 1.5)$ (c) $(R_{\theta_1}, R_{\theta_2}) = (1.25, 0.75)$ (d) $(R_{\theta_1}, R_{\theta_2}) = (2, 2)$

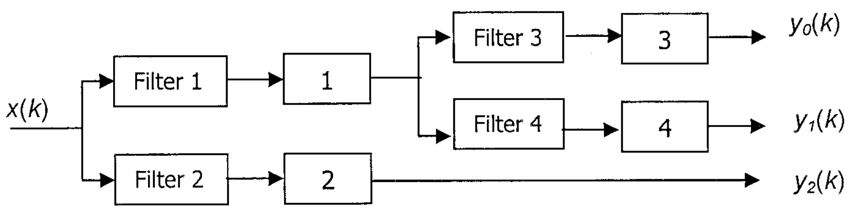
SUBBAND CODING

A25. Statement I : In subband coding: the sum of the subbands' quantization error is always equal to the overall quantization (or coding) error.

Statement II : In DCT coding: The sum of the DCT coefficients' quantization error is equal to the overall quantization (or coding) error.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A26. Consider the following block diagram describing a subband decomposition:

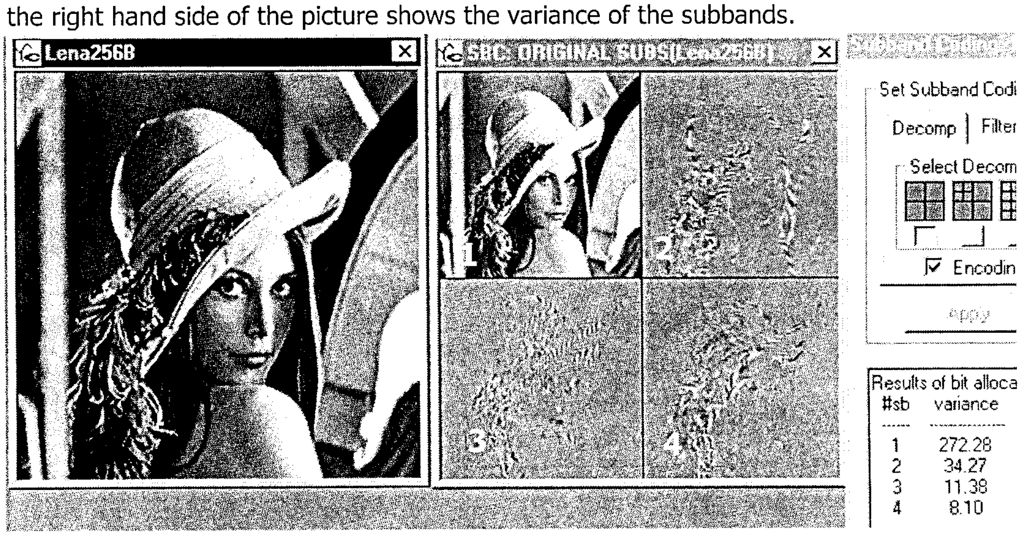


Statement I : If 'Filter 1' and 'Filter 2' are QMF filters, then the box numbered '1' is a 2:1 decimation.

Statement II : Only if 'Filter 1' and 'Filter 2' are ideal low-pass and high-pass filters, respectively, then $y_2(k)$ contains only those frequency components of $x(k)$ that reside in the frequency band $[-\pi,-\frac{\pi}{2}]\cup[\frac{\pi}{2},\pi]$ in $x(k)$.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A27. Given is the following decomposition of the "Lena" image into four subbands. The text pane on the right hand side of the picture shows the variance of the subbands.



Each of the subbands will be PCM + Huffman encoded, such that the average bit rate is 1.5 bit per pixel

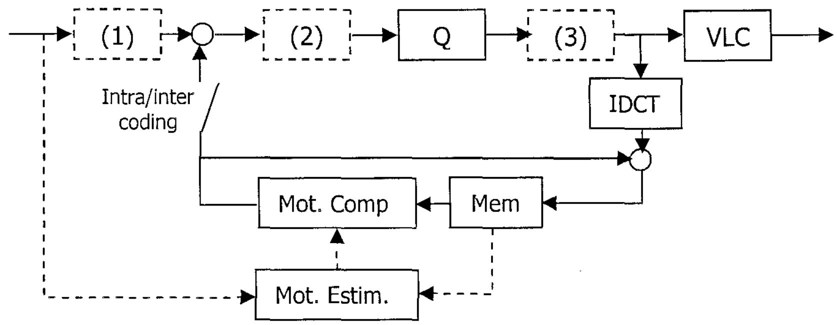
Statement I : Subband 2 will be encoded at a bit rate of 1.59 bit per subband pixel

Statement II : Subband 3 will be encoded at a higher bit rate than subband 2.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

VIDEO COMPRESSION

A28. Consider the following block diagram describing a video compression system:



Statements:

(I) For efficient compression, the DCT should be at position (2).
(II) For efficient compression, the DCT should be at position (3).
(III) Putting the DCT at positions (1) yields a proper motion-compensated DCT-based video compression scheme.
(IV) Putting the DCT at positions (1) or (2) yields equivalent compression systems if intra-frame coding is applied.

Answers:
(a) All statements are correct
(b) Only II is correct
(c) Only I and IV are correct
(d) Only I and III are correct

A29. In MPEG, the group of pictures (GOP) structure is used.

Statement I : B-frames can be skipped by the decoder without affection the decoded quality of the subsequent I and P frames.
Statement II : A GOP may exist of I frames only.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A30. In MPEG, the following group of pictures (GOP) structure is used. I B P B B P P B B B P

Statement I : The decoder receives the following order of frames: I P B P P B B P B B B
Statement II : A decoder can skip the decoding of the B frame as long as it decodes all subsequent P frames (four in this example)

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

B. Open Questions Section (6 questions)

B1. Give the definition of signal-to-noise ratio (SNR) of a scalar quantizer. Explain what the different symbols in the SNR definition mean, or give a definition of the symbols in mathematical terms.

B2. Encode the following sequence using the LZ78 approach:

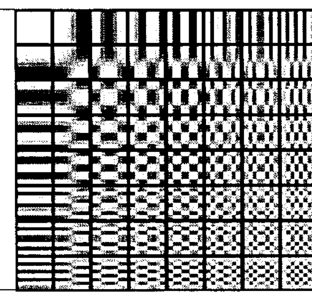
d o b e e d o o b a d e e

Show the generated dictionary, and the encoder output.

B3. Design a Huffman variable length code for a six-level non-uniform symmetric scalar quantizer for which the following probabilities of the representation levels Q(X) are given:

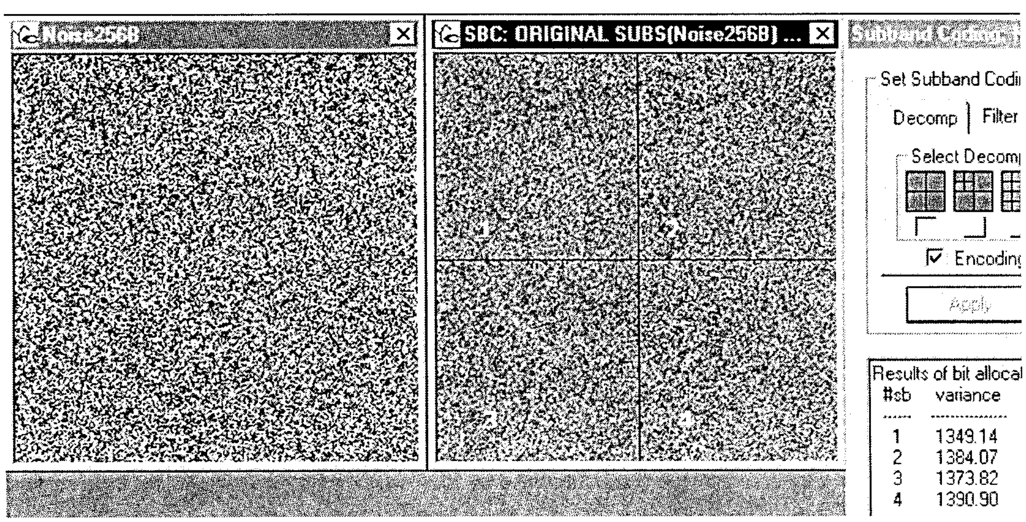
P[Q(X)=0.55]=0.35 P[Q(X)=1.20]=0.10 P[Q(X)=2.45]=0.05

B4. Explain in which way an 8x8 block of pixels is represented using the DCT basis functions $t_{k,l}$ shown in the following picture:



B5. We consider motion estimation using block matching. The block size is 8x8, and the maximum motion vector length is 6 horizontally and 6 vertically. Calculate the number of motion vectors that needs to be evaluated per 8x8 block for full (brute force) search and for the one-at-a-time search.

B6. Given is the following decomposition of the "noise" image into four subbands. The text pane on the right hand side of the picture shows the variances of the subbands.



Two researchers have a debate about the usefulness of subband coding the above "noise" image. **Researcher R.** claims that subband coding is useful even for noisy images because the subbands may have different variances. **Researcher J.** claims that subband coding of noisy data is always useless because noise is uncorrelated by definition. **Defend one of the two opinions with arguments.**