DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics, and Computer Science
Information and Communication Theory Group

EXAMINATION MULTIMEDIA COMPRESSION

Wednesday November 1, 2006 (9 am –12 noon)

The questions are posed in English. Answers to open questions (Section B) can be either in Dutch or English, depending on your preference. Keep your answers as concise as possible. Answers to the multiple choice questions (Section A) should be written on a separate sheet of paper.

This exam has an open question and a multiple choice section

A. Multiple Choice Questions Section (28 questions)

Evaluate the statements given, or find the question to the posed question(s), then select the correct answer (a, b, c, or d) and fill out the answer on the answer form. Do not forget to hand-in this answer form.

A1. Statement I : Variable length encoding is a reversible coding technique.
Statement II : Quantization is a reversible coding technique.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A2. Statement I : Uncorrelated signals can not be compressed losslessly.
Statement II : With lossy compression *any* compression factor is attainable.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A3. We consider the Lempel-Ziv (LZ78) coding of the following bit string:

0 0 1 0 1 0 1 1 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 0 0

Statement I : The LZ78 coding of this bit string yields a partitioning into 10 substrings (bit patterns or dictionary entries)
Statement II : The last substring (bit pattern or dictionary entry) is "000"

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A4. Statement I : The entropy of a scalar quantizer's representation levels is always smaller than or equal to the average codeword length of a Huffman code.
Statement II : The average codeword length of a Huffman code of a symmetric scalar quantizer with 3 or more representation levels is always larger than 1 bit per symbol.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A5. Given is the following 5-level scalar quantizer, designed for a signal with $\mu = 0$ and $\sigma = 1$:

Number	Representation Level	Probability
1	-2.25	0.055
2	-0.84	0.221
3	0.00	0.448
4	0.84	0.221
5	2.25	0.055

The entropy $H(X)$ of this quantizer is equal to:

Answers:
(a) 0.585 bit/quantizer level
(b) 1.346 bit/quantizer level
(c) 1.942 bit/ quantizer level
(d) 2.322 bit/ quantizer level

A6. Given is the 5-level scalar Lloyd-Max quantizer designed for a signal with $\mu = 0$ and $\sigma = 1$:

Number	Representation Level	Probability
1	-2.25	0.055
2	-0.84	0.221
3	0.00	0.448
4	0.84	0.221
5	2.25	0.055

Statement I: The quantizer has a decision threshold at –0.42.
Statement II: The quantizer is a uniform quantizer.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A7. Given are the four VLC codes for a 5-level scalar quantizer designed for a signal with $\mu = 0$ and $\sigma = 1$:

Number	Representation Level	Probability	VLC code 1	VLC code 2	VLC code 3	VLC code 4
1	-2.25	0.055	001	0010	0010	00
2	-0.84	0.221	01	000	01	10
3	0.00	0.448	1	1	1	0
4	0.84	0.221	0001	01	001	01
5	2.25	0.055	0000	0011	0000	11

The uniquely decodable VLC code that yields the shortest average codeword length is:

Answers:
(a) VLC code 1
(b) VLC code 2
(c) VLC code 3
(d) VLC code 4

A8. Given is the following 5-level (prototype) scalar quantizer designed for a signal with $\mu = 0$ and $\sigma = 1$:

Number	Representation Level
1	-2.25
2	-0.84
3	0.00
4	0.84
5	2.25

The variance of the signal to be quantized is $\sigma = 2$. In order to match the prototype quantizer to this signal, the quantizer decision and representation levels are scaled.

Statement I: The scaled quantizer has a representation level at 9.00

Statement II: If $H_p(X)$ is the entropy of the above prototype quantizer, the scaled quantizer has entropy $H_p(X) + \log_2(\sigma^2) = H_p(X) + 2$

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A9. We consider the DPCM coding of a Gaussian distributed signal $x(k)$ with the autocorrelation function $R_x(k)$. The variance of $x(k)$ is $\sigma_x^2 = 1600$, and the variance of the prediction difference $\Delta x(k)$ is $\sigma_{\Delta x}^2 = 100$.

Statement I: The theoretically expected gain of DPCM over PCM is 4 bit/sample.

Statement II: The larger the variance of a signal is, the larger the prediction gain will be.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A10. Given is a signal $x(k)$ with the following autocorrelation function:

$$R_x(k) = 20\ (0.8)^{|k|}$$

The signal $x(k)$ is encoded using DPCM. The DPCM system makes use of a first order linear predictor with prediction coefficient h_1.

The optimal prediction coefficient h_1 is equal to

Answers:
(a) $h_1 = 0.0$
(b) $h_1 = 0.2$
(c) $h_1 = 0.8$
(d) $h_1 = -0.2$

A11. Given is a signal $x(k)$ with the following autocorrelation function:

$$R_x(k) = 20\ (0.8)^{|k|}$$

The signal $x(k)$ is encoded using DPCM. The DPCM system makes use of a first order linear predictor.

The prediction gain G_p is equal to:

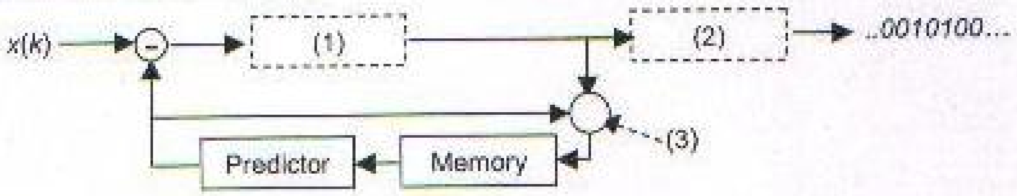
Answers:	(a) $G_p = 1$	(c) $G_p = 5$
	(b) $G_p = \frac{25}{9}$	(d) $G_p = 20$

A12. Statement I : For uncorrelated signals, the prediction gain can be made larger than 1 by choosing the proper order for the linear predictor.

Statement II : For uncorrelated signals, the optimal prediction coefficients h_k are all equal to 1.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A13. Given is the following DPCM system



Answers:
(a) The quantizer should be at position (1), and the operation at (3) should be addition (+)
(b) The quantizer should be at position (2), and the operation at (3) should be addition (+)
(c) The quantizer should be at position (1), and the operation at (3) should be subtraction (-)
(d) The quantizer should be at position (2), and the operation at (3) should be subtraction (-)

A14. Given are the following two-dimensional predictors (the index i indicates the line number, the index j the column number in the picture):

Predictor 1: $x(i,j) = 0.2x(i,j-1) + 0.2x(i,j+1) + 0.2x(i-1,j) + 0.2x(i+1,j)$

Predictor 2: $x(i,j) = 0.4x(i-1,j-1) + 0.3x(i-1,j)$

Statement I : Predictor 1 can be used for DPCM encoding of images.
Statement II : Predictor 2 can be used for DPCM encoding of images.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A15. We consider an image for which the following values of the 2-dimensional autocorrelation function $R_x(i,j)$ have been calculated

$$R_x(i,j) = \begin{bmatrix} R_x(-1,-1) & R_x(-1,0) & R_x(-1,1) \\ R_x(0,-1) & R_x(0,0) & R_x(0,1) \\ R_x(1,-1) & R_x(1,0) & R_x(1,1) \end{bmatrix} = \begin{bmatrix} 8.27 & 8.7 & 8.27 \\ 9.5 & 10.0 & 9.5 \\ 8.27 & 8.7 & 8.27 \end{bmatrix}$$

If we use the following 2-D predictor in a DPCM system:

$$x(i,j) = \alpha x(i,j-1) + \beta x(i-1,j)$$

then the optimal prediction coefficients α and β are equal to:

Answers:
(a) $\alpha = 0.870$ and $\beta = 0.950$
(b) $\alpha = 0.822$ and $\beta = 0.155$
(c) $\alpha = 0.729$ and $\beta = 0.267$
(d) $\alpha = 0.155$ and $\beta = 0.822$

A16. Given is the following decorrelating transform T for the case N=4:

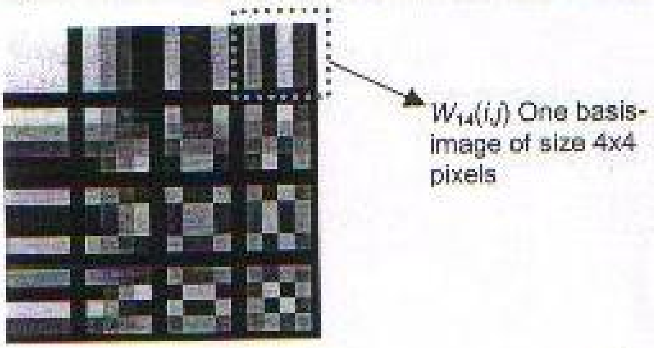
$$T^t = \begin{bmatrix} \frac{1}{2} & \frac{3}{\sqrt{10}} & \frac{1}{2} & \frac{1}{\sqrt{10}} \\ \frac{1}{2} & \frac{1}{\sqrt{10}} & -\frac{1}{2} & -\frac{3}{\sqrt{10}} \\ \frac{1}{2} & -\frac{1}{\sqrt{10}} & -\frac{1}{2} & \frac{3}{\sqrt{10}} \\ \frac{1}{2} & -\frac{3}{\sqrt{10}} & \frac{1}{2} & -\frac{1}{\sqrt{10}} \end{bmatrix}$$

Statement I : The vector $\left(\frac{1}{2}, \frac{3}{\sqrt{10}}, \frac{1}{2}, \frac{1}{\sqrt{10}}\right)$ is a basis vector of this transform.

Statement II : The transform T is not orthogonal.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A17. Given are the following 16 basis-images $W_{kl}(i,j)$ used in transform coding of images with a block size of 4x4.

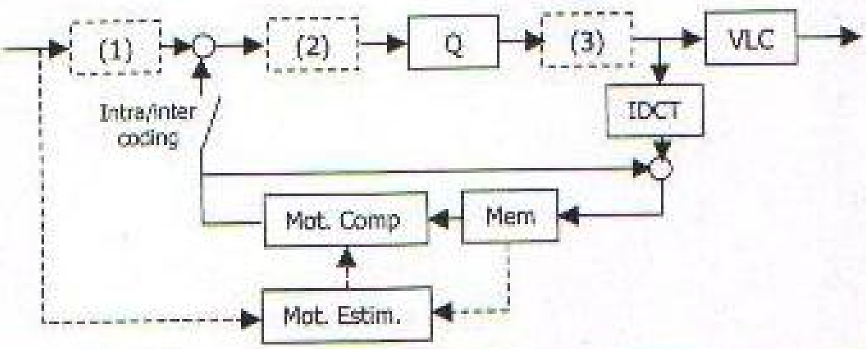


Statement I : All of these basis-images have to be mutually orthogonal.

Statement II : If θ_{kl} is the transform coefficient that belongs to basis-images $W_{kl}(i,j)$, then for *any* image θ_{kl} =-1, θ_{kl} =0, or θ_{kl} =1.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A18. Consider the following block diagram describing a video compression system:



Statements:

(I) For efficient compression, the DCT should be at position (2).
(II) For efficient compression, the DCT should be at position (3).
(III) Putting the DCT at positions (1) yields a proper motion-compensated DCT-based video compression scheme.
(IV) Putting the DCT at positions (1) or (2) yields equivalent compression systems if intra-frame coding is applied.

Answers:
(a) All statements are correct
(b) Only II is correct
(c) Only I and IV are correct
(d) Only I and III are correct

A19. Given is a signal $x(k)$ with zero-mean, of which the following autocorrelation coefficients are given:

$$R_x(0) = 6.25,\ R_x(1) = 4.75,\ R_x(2) = 3.25,\ R_x(3) = 2.75$$

The following 4x4 transform is applied to $x(k)$:

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

Statement I : The resulting (four) transform coefficients are uncorrelated.
Statement II : The variance of the first transform coefficients is equal to 18.0.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A20. Statement I: The efficiency of DCT-based image compression is independent of the block size (e.g. 4x4, 8x8).

Statement II: The DCT is a powerful correlation-reducing transform, but it is not identical to the optimally decorrelating transform.

Answers:
(a) I and II are both correct
(b) Only I is correct
(c) Only II is correct
(d) Neither I nor II is correct

A21. Given is a signal $x(k)$ with zero-mean, of which the following autocorrelation coefficients are given: $R_x(0) = 5$ and $R_x(1) = 3$. Consider the following decorrelating transform:

$$\mathbf{T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The transform is applied to the signal $x(k)$, yielding the (sets of) decorrelated transform coefficients θ_1 and θ_2. We wish to encode the signal $x(k)$ at an average bit rate of 2 bit per sample. The optimal bit allocation result for θ_1 and θ_2, denoted as $\left(R_{\theta_1}, R_{\theta_2}\right)$, is

Answers:
(a) $\left(R_{\theta_1}, R_{\theta_2}\right) = (4\ ,\ 0)$
(b) $\left(R_{\theta_1}, R_{\theta_2}\right) = (3\ ,\ 1)$
(c) $\left(R_{\theta_1}, R_{\theta_2}\right) = (2\ ,\ 0)$
(d) $\left(R_{\theta_1}, R_{\theta_2}\right) = (2\ ,\ 2)$

A22. We apply vector quantization to a 1-D signal $x(k)$. The code book is given by the following four code book vectors:

$c_1 = (1.5, 3)$ $c_2 = (-1, -2)$ $c_3 = (0, 5)$ $c_4 = (-3, 1)$

The average bit rate at which $x(k)$ will be compressed is equal to

Answers:
- (a) 2 bit / sample
- (b) 1 bit / sample
- (c) 0.5 bit / sample
- (d) Not enough information to answer this question

A23. We apply vector quantization to a 1-D signal $x(k)$. The first 8 samples of the signal $x(k)$ are given by:

$x(0)$	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$	$x(7)$
4	-3	2	1	-3	0	5	0

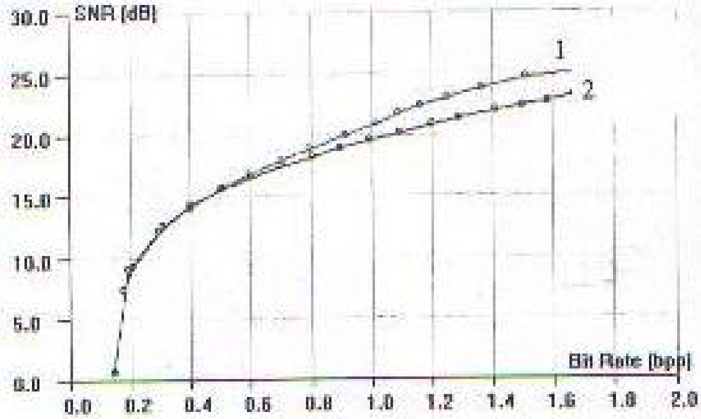
The code book is given by the following four code book vectors:

$c_1 = (1.5, 3)$ $c_2 = (-1, -2)$ $c_3 = (0, 5)$ $c_4 = (-3, 1)$

The encoder uses the mean square error criterion in the encoding process. The encoder produces the following sequence of code book addresses:

Answers:
- (a) 2 – 4 – 2 – 3 – 1
- (b) 2 – 4 – 1 – 2 – 4 – 3 – 1
- (c) 2 – 1 – 4 – 1
- (d) None of the answers (a)-(c) are correct

A24. The following figure shows the bit rate-SNR performance for the JPEG compression of the 'Lena' image using a different normalization (quantization or weighting) matrix. In one case the standard JPEG normalization matrix has been used, in the other case no normalization matrix (or similarly, a normalization matrix filled with identical values) has been used.



Statement I : Curve "1" is the result of JPEG using the standard JPEG normalization matrix.

Statement II : The answer to the question "which of the two normalization matrices yields the best bit rate-SNR performance" is image dependent.

Answers:
- (a) I and II are both correct
- (b) Only I is correct
- (c) Only II is correct
- (d) Neither I nor II is correct

A25. The picture below shows an compressed (and decompressed) image that has been received with bit errors in the compressed bit stream.

The compression technique used is

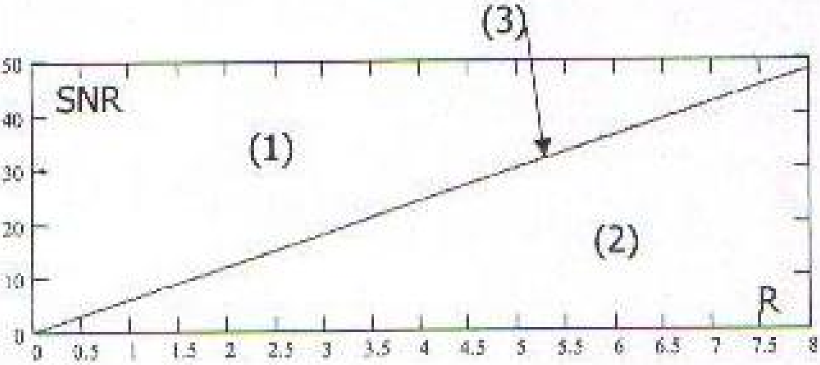
Answers:

(a) DPCM
(b) Vector quantization
(c) Transform coding
(d) Subband coding

A.26. We consider the compression of a memoryless Gaussian distributed signal *X*. It is known that the relation between bit rate and distortion is described by the following rate-distortion function:

$$R = \frac{1}{2}\log_2\left(\frac{\sigma_x^2}{\sigma_q^2}\right).$$

Using this function we create the following bit rate versus SNR graph:



Statements:

(I) Any scalar quantizer has a performance in area (2). A scalar quantizer exists that has a performance arbitrarily close to the line indicated by (3).

(II) Any compression system has a performance that lies in area (2). A compression system exists that has a performance arbitrarily close to the line indicated by (3).

(III) Any scalar quantizer has a performance in area (1). A scalar quantizer exists that has a performance arbitrarily close to the line indicated by (3).

(IV) Any compression system has a performance that lies in area (1). A compression system exists that has a performance arbitrarily close to the line indicated by (3).

Answers:

(a) Only II is correct
(b) Only III and IV are correct
(c) Only I and III are correct
(d) Only III is correct

A27. The quantization error σ_q^2 of any 4x4 image block $x(i,j)$ that is subject to a 4x4 DCT transform can be written as follows:

$$\sigma_q^2 = \sum_{i=1}^{4}\sum_{j=1}^{4}\left[\ldots - \sum_{k=1}^{4}\sum_{l=1}^{4}\hat{\theta}_{kl}W_{kl}(i,j)\right]^2$$

where $\hat{\theta}_{kl} = Q(\theta_{kl})$ are quantized DCT coefficients and $W_{kl}(i,j)$ are the DCT basis-images.

At the position of the dots (...) in the above equation, we need to fill in:

Answers:

(a) θ_{kl}

(b) $x(i,j)$

(c) $\theta_{kl}W_{kl}(i,j)$

(d) The entire equation is incorrect

A28. We consider motion estimation using block matching. The block size is 8x8, and the maximum motion vector length is 4 horizontally and 4 vertically. Calculate the number of motion vectors that needs to be evaluated per 8x8 block for full (brute force) search (FS) and one-at-a-time search (OTS):

Answers:

(a) FS: 25 OTS: 17

(b) FS: 81 OTS: 9

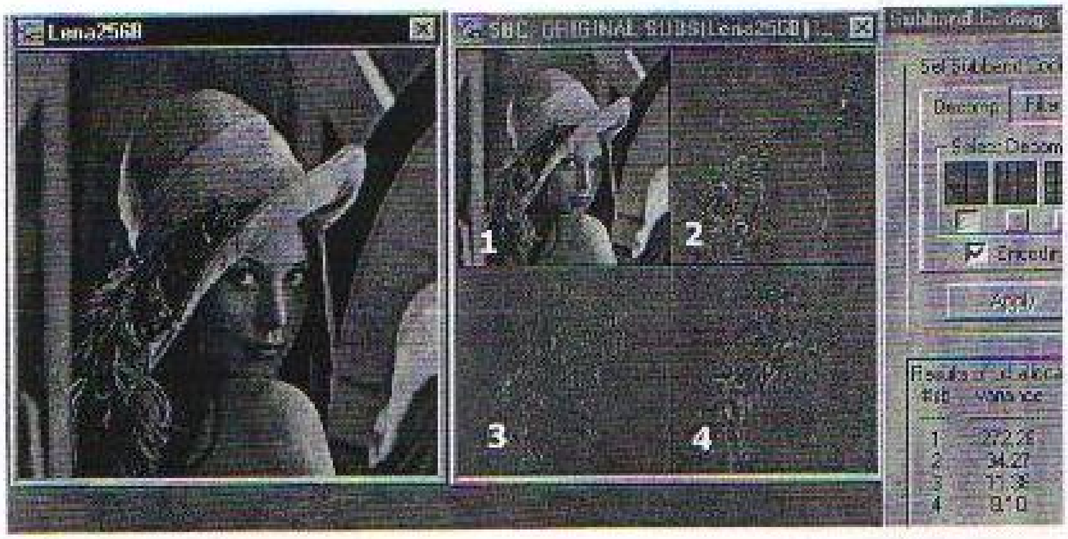
(c) FS: 81 OTS: 17

(d) FS: 25 OTS: 9

B. Open Questions Section (6 questions)

B1.	Sketch the basic two-channel filter bank used in subband coding of signals, and name all components.
B2.	A basic two-channel filter bank can be used as building block to decompose a signal into more than two subbands. Explain this principle and give as an example the decomposition structure used in wavelet coding.
B3.	In subband coding, the low-pass and high-pass filters used in the two-channel filterbank are not ideal. Therefore, both the low-pass filtered and high-pass filtered subbands of the signal $X(\omega)$ contain a significant amount of aliasing i.e. $X(\omega+\pi)$. Explain in which way the overall transfer function of the two-channel filterbank can still be alias free, assuming that the subbands are not quantized.

B4. Given is the following decomposition of the "Lena" image into four subbands. The text pane on the right hand side of the picture shows the variance of the subbands.



Each of the subbands will be PCM encoded. The bit allocation procedure yields the following optimally allocated bit rates:

Subband 1:	2.58 bpp	Subband 3:	0.28 bpp
Subband 2:	1.09 bpp	Subband 4:	0.05 bpp

Calculate the average bit rate, and discuss the problem(s) with the above result of the bit allocation.

B5.	Estimate the overall distortion of the image shown above when using the given bit allocation result and assuming that the subband data is Gaussian distributed.
B6.	Explain in what way a motion-compensated video encoder exploits inter-frame and intra-frame correlations in video sequences. Describe the principle of motion estimation (by hierarchical block matching) within motion-compensated video compression.