DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Information Technology and Systems
Information and Communication Theory Group

EXAM DIGITAL SIGNAL CODING

Thursday October 31, 2002 (2 pm – 5 pm)

This exam consists of five (5) questions.

The questions are posed in English. Answers can be either in Dutch or English, depending on your preference. ***Please answer each question on a separate piece of paper****, and put your name on each page. Success!*

Question 1: (a) 1; (b) 2; (c) 1; (d) 1; (e) 1; (f) 2; (g) 1; (h) 2; (i) 2

Given is the following prototype (7-level) Lloyd-Max scalar quantizer:

Number	Representation Level	Probability
1	-3.525	0.0149
2	-1.643	0.0713
3	-0.636	0.1964
4	0.000	0.4348
5	0.636	0.1964
6	1.643	0.0713
7	3.525	0.0149

The variance of the quantization error is $\sigma_q^2 = 0.0828$. This 7-level non-uniform quantizer has been optimized using the Lloyd-Max (iterative) design procedure assuming that the quantizer input is distributed according to the PDF $f_X(x)$.

(a) Explain which criterion is minimized in the Lloyd-Max design procedure (it is *not* necessary to describe the procedure itself, only the minimization criterion should be given).

(b) Calculate the quantizer decision thresholds, and sketch the quantizer characteristic.

(c) Calculate the performance of the scalar quantizer in terms of signal-to-noise ratio (SNR, in dB).

(d) Calculate the entropy $H(X)$ of the (output of the) scalar quantizer.

(e) Determine the maximal compression factor that can be achieved if the representation levels are encoded using *entropy coding*.

(f) Design a Huffman VLC for the quantizer.

(g) Calculate the average code word length of the Huffman code designed under question (f).

(h) We quantize a signal with zero-mean and variance $\sigma_X^2 = 36$. Determine the quantizer representation levels for this signal. Determine the performance of the quantizer in terms of the overall quantization error variance ánd the signal-to-noise ratio (in dB).

(i) Determine if the following statements are true or not (assuming the same PDF), and explain why:
1. *No other 7-level prototype quantizer* exists that has a smaller quantization error variance than the one given in the table above.
2. A prototype 7-level *uniform* quantizer that can be designed for the PDF $f_X(x)$ has a smaller quantization error variance than the one given in the table above.
3. *No quantizer with entropy H*(*X*) (calculated under question (d)) exists that has a *smaller quantization error variance* than the one given in the table above.

Question 2: (a) 2; (b) 1; (c) 3; (d) 1; (e) 2

Let us assume that in a subband coding system we have 3 subbands of equal size (same number of pixels in each of the subbands). The subbands have zero-mean. The autocorrelation function of the three subbands is given by:

Subband	**Autocorrelation function**
1	$R_X(k) = 100\,(0.5)^{\lvert k \rvert}$
2	$R_X(k) = 10\,(0.95)^{\lvert k \rvert}$
3	$R_X(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$

Subbands can be encoded by PCM or by first-order DPCM.

(a) Let us consider DPCM encoding of the three subbands. Determine the optimal prediction coefficient h_1 for the three subbands (note, the three subbands each have a different prediction coefficient).

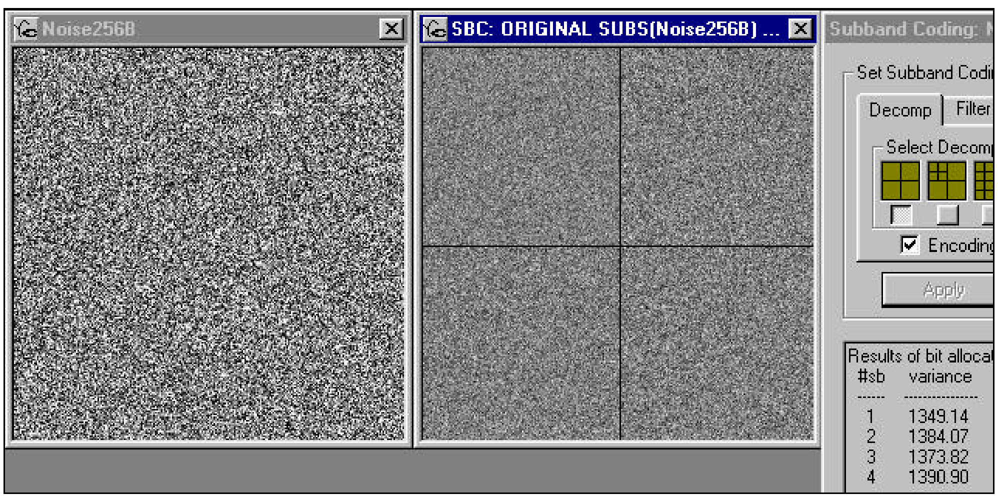
(b) Determine the prediction gain for the three subbands, and give a motivated recommendation for the coding of the three subbands (PCM versus first-order DPCM).

In the following we simply assume that all subbands are PCM encoded. We have the choice of four prototype quantizers with the following performance:

Quantizer	R	$?_q^2$
Q_0	0.0	1.000
Q_1	1.0	0.300
Q_2	2.0	0.060
Q_4	4.0	0.005

(c) Determine the optimal bit allocation under the constraint that the average bit rate is 4/3 bit per sample (hint: there are only 9 different bit allocations that satisfy this bit rate constraint).

(d) Consider the subband decomposition of the noise image shown below. Assume that we wish to encode this image with an average bit rate of 1 bit per pixel. What is the optimal bit allocation? Explain your answer.



(e) For a white noise image any subband decomposition (including no decomposition) will *theoretically* yield the same overall SNR performance (of course when the average bit rate is the same). Explain why this so.

Question 3: (a) 2; (b) 2; (c) 1; (d) 1; (e) 2

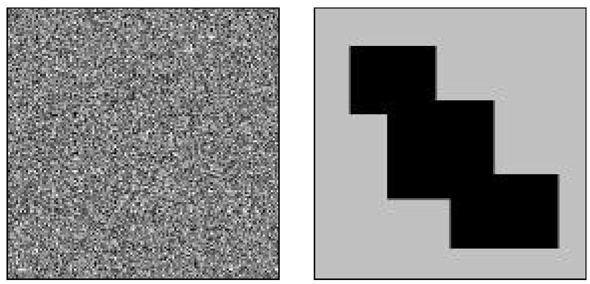
(a) Sketch the block diagram of an image vector quantization compression system, and name all components.

(b) Explain the principle of vector quantization. Use in your explanation at least the following terms: vector, codebook, codebook-vector, codebook dimension, bit rate.

(c) In vector quantization of images, if the size of the codebook vectors is 4x4 and the codebook has 8 vectors, what is the bit rate of this compression system in terms of the number of bits per pixel? Explain your calculation briefly.

(d) In vector quantization, we use *training-vectors* to design the codebook, whereas in scalar quantization we use the *probability density function* of the signal to be quantized. Motivate the use of training-vectors rather than probability density functions in the design of vector quantization codebooks.

(e) We use the following training pictures (of size 256x256) to design a VQ codebook that contains 3 *codebook vectors* of size 4x4. Explain which vectors you would expect to show up in this codebook, and motivate your answer.



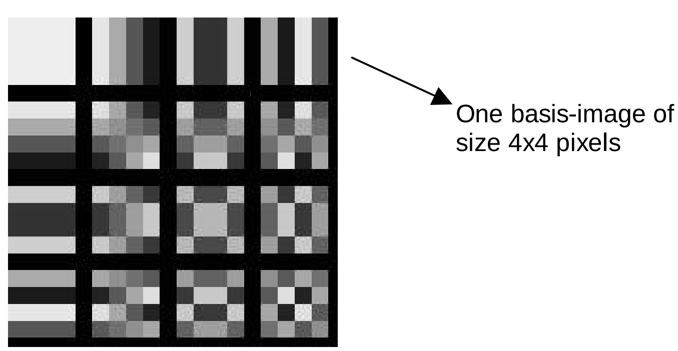
Question 4: (a) 2; (b) 2; (c) 2; (d) 2; (e) 2; (f) 2

This question concerns DCT-based coding, on which for instance JPEG is based.

(a) The DCT is an orthogonal transform. Explain what we mean by orthogonality of the transform.

(b) There is a compelling reason from quantization and bit allocation perspective to use only orthogonal transforms – such as the DCT – in transform coding. Give this reason.

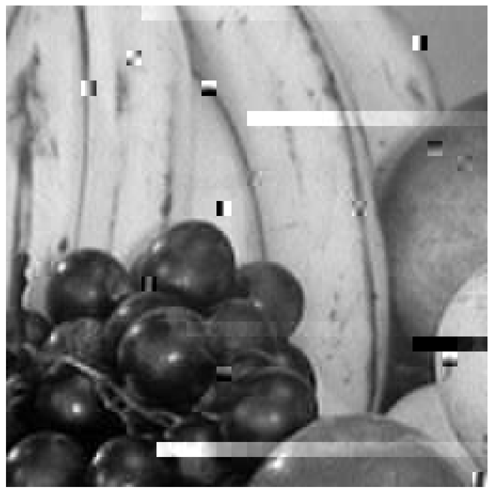
Let us assume that we apply the DCT transform on 4x4 subblocks of the image to be encoded. In the picture below the 16 basis-functions (~basis-images) are illustrated.



(c) Explain the relation between the (above) DCT basis-images, the DCT coefficients, and the original image subblock.

(d) After quantization, many of the quantized DCT-coefficients are equal to zero. Explain how zig-zag scanning of the DCT coefficients and run-length coding can be used for efficient representation of the zero-valued DCT-coefficients.

The picture below shows a DCT-encoded (and decoded) image that has been received with bit errors in the compressed bit stream.



(e) Describe how bit errors result in the degradations that you observe in this decoded picture.

(f) Sketch the block-diagram of the JPEG encoder and decoder, name its components, and describe the role of the quantization (or normalization) matrix.

Question 5: (a) 2; (b) 1; (c) 3; (d) 2;

We consider the compression of a short video sequence of which the frames are printed on the last page of this exam. Video sequences have a lot of temporal correlation. For that reason temporal DPCM (or inter-frame compression) is very efficient.

(a) Explain the principle of temporal DPCM.

(b) Give one reason why usually a *first-order* temporal predictor is used.

(c) Sketch the block diagram of the motion-compensated (hybrid) video coding system and explain why:

- ?? Motion estimation is included in the encoder.
- ?? Spatial forward DCT is included in the encoder.

?? The spatial inverse DCT has to be included in the encoder.
?? The decoder does not need to carry out motion estimation.

For the video sequence shown here, we have measured the following values for the variance of the *frames*, variance of the *frame differences*, and variance of the *motion compensated frame differences*:

Frame number	Variance of frame	Variance of frame difference	Variance of motion compensated frame difference
2	4581.6	3397.5	650.4
3	4554.9	3269.3	494.9
4	4597.1	3594.1	555.0
5	4819.5	4304.4	872.0
6	4877.9	4367.8	944.2
7	4859.5	3866.7	685.6
8	4741.5	3463.3	511.2
9	4613.4	3859.8	401.7
10	4427.9	4096.9	463.2
11	3701.9	6386.4	3416.9
12	3707.4	705.7	170.7
13	3777.0	444.1	312.1
14	3782.8	291.0	170.5

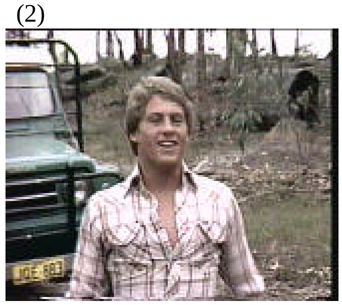
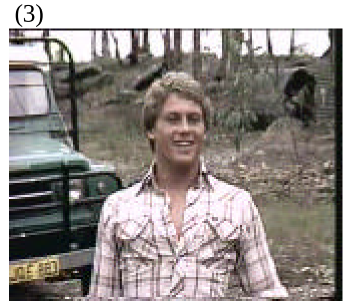
(d) The above numbers can be used to decide which type of compression is optimally suited for each frame:
?? intra-frame coding,
?? inter-frame coding (no motion compensation),
?? motion-compensated inter-frame coding.

Which choices would you expect a well-designed video encoder (such as an MPEG encoder) to make for the encoding of the individual video frames? Motivate your answer with numerical arguments!

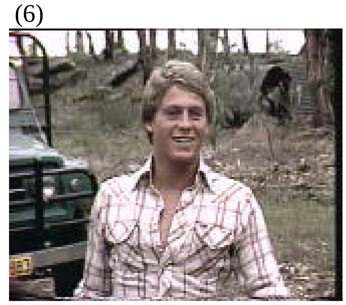

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

(12)

(13)

(14)

Good luck with the Exam!