Technische Universiteit Delft
Fac. Elektrotechniek, Wiskunde en Informatica

Examination Valuation of Derivatives, Wi 3405TU

Friday January 27nd 2016, 9:00 - 11:00 **(2 hours examination)**

1. Consider the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0,$$

with a payoff function as the final condition at $t = T$.

   a. Show that the transformations $S = e^y$, $\tau = T - t$, and $v(y, \tau) = e^{r\tau}V(y, \tau)$, followed by $x = y + (r - \frac{1}{2}\sigma^2)\tau$ result in the following heat equation for unknown $u(x, \tau)$,

$$\frac{\partial u}{\partial \tau} = \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2},$$

   b. Write down the Forward-in-Time Central-in-Space scheme, FTCS, for this heat equation. Show that the discrete scheme is second-order accurate in space and first-order accurate in time.

   c. Apply von Neumann stability analysis for this heat equation. Show that the stability condition takes the form $\sigma^2 \Delta\tau \leq (\Delta x)^2$.

2. A double barrier option has two barriers, B_1 and B_2, one above and one below the current stock price, $B_1 < S_0 < B_2 < E$, with strike price E. In a *one-touch double barrier put option*, the option is knocked out, resulting in a valueless option, if at least one of the barriers is breached during the life of the option.

We assume that the asset follows the process that leads to the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0.$$

   a. Why would an investor buy such an option, and why would a writer sell such an option?

   b. Transform the equations by $\tau = T - t$ into forward equations in time. Give the appropriate computational domain, the corresponding boundary conditions and the payoff function for the one-touch double barrier put option.

   c. Which standard (single) barrier options should be in a portfolio with this double barrier option in order to replicate a regular European put option?

   d. Why does it not make much sense to have $B < E$, with E the strike price, in an up-and-in barrier call option?

Z.O.Z.

3. The following Matlab code is given:

```
%%%%%%%%%% Problem and method parameters %%%%%%%%%%%%%
S = 3; E = 2; T = 1; r = 0.05; sigma = 0.3;
M = 400; dt = T/M; p =0.5;
u = exp(sigma*sqrt(dt) + (r-0.5*sigma^2)*dt);
d = exp(-sigma*sqrt(dt) + (r-0.5*sigma^2)*dt);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Time T option values
W = max(E-S*d.^([M:-1:0]').*u.^([0:M]'),0);

% Work back to option value at time zero
for i = M:-1:1
    W = exp(-r*dt)*(p*W(2:i+1) + (1-p)*W(1:i));
end

disp('Option value is'), disp(W)
```

- What is the computational technique in this Matlab code and what is computed? Adapt the Matlab code above to value a Bermudan put option with two early-exercise dates, at $t = 0.5$ and at $T = 1$.

Place your name and study number on each page with solutions.