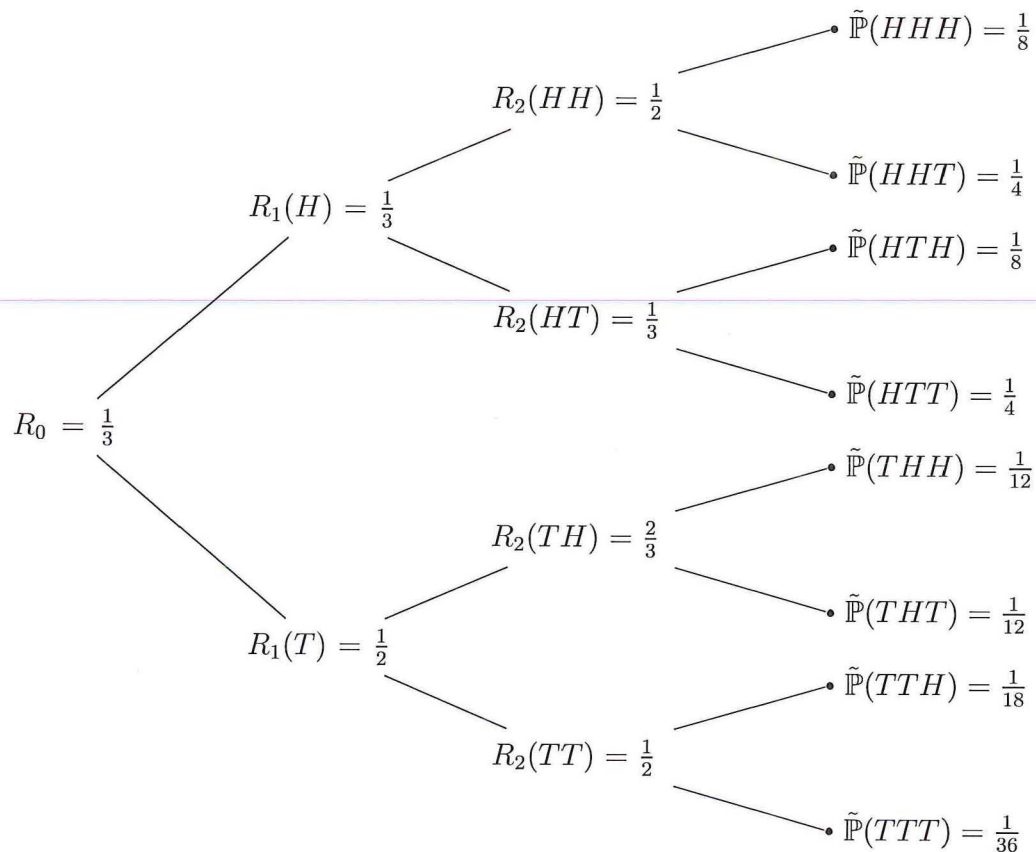


Introduction to Mathematical Finance (wi3417tu) January 31st 2017, 18.30–21.30

(No books, no notes.)

Please note: answers should be supplemented by motivation, explanation and/or calculation, whichever may be appropriate; you may choose Dutch or English as the language to use for your answers. **Point distribution:** each *part* of a question is worth 1 point; the grade is the number of points earned plus 1.

1. Write down all the valid *stopping times* for the 1-period binomial model (number them τ_1 , τ_2 , etcetera).
2. Consider the following 3-period binomial interest rate model with risk-neutral probabilities:



- a. Construct a table with the distributions of D_1 , D_2 , and D_3 . Also determine all transition probabilities in the above tree.
- b. Determine $B_{0,2}$ and $B_{1,3}$.
- c. Explain what a *3-period interest rate cap* is. Determine the price if $K = 1/3$ is the maximal interest rate to be paid.

3. Consider a 3-period binomial model for the stock price-evolution with $u = 2$, $d = 1/2$, $r = 1/4$ and $S_0 = 12$.
- a. Determine the time-zero value of an American put option with expiration $n = 3$ and strike 9. Also, specify the optimal exercise rule.
 - b. Suppose you are holding one such option and at time $n = 1$ someone offers to exchange it for an American call with the same expiration and the same strike. Should you accept it?
 - c. After the first coin toss, which yields $\omega_1 = T$, it becomes known that the interest rate goes up. Will this affect your decision under **b.**? If so, how? N.B. Only consider the case $\omega_1 = T$.
4. Consider the N -period binomial interest rate model, $0 \leq n \leq m \leq N - 1$.
- a. A derivative pays at time $m + 1$: $V_{m+1} = R_m$. Express the time- n price V_n in terms of zero-coupon bond prices.
 - b. Explain the meaning of $F_{n,m}$ and specify the details of the related contract. Does $F_{n,m}$ equal V_n ? Explain why (not).