Exam Risk 2015

1) Consider a function $C(u,v)=uv(1+\alpha(1-u)(1-v))$, $0<u, v<1$.
   a) For which values of α is C a copula cdf?
   b) Find the relationship between the correlation and the parameter of this copula.
   c) How can we draw samples from this copula?

2) Assume a product is built on an assembly line. The following data is available describing the number of units produced before the first defective unit is produced per day

day	Nr of units produced before defective is produced
1	1
2	1
3	6
4	7
5	2

   a) Fit the geometric distribution to data above (Find parameter p).
   b) Consider the uniform distribution on the interval (0,1) as a prior distribution of the parameter p of the geometric distribution. Calculate the posterior distribution given data observed in the table above.
   c) Compare the prior and the posterior means for p.

3) It has been discovered that the chance that a person will commit a credit-card fraud does not depend on the sex of a person. However after the fraud is committed male and female will behave differently. If a male commits a credit-card fraud then the chance that he will buy gas within 24 hours from the fraud increases 20 fold as compared to the case when he does not commit the fraud. Committing a credit-card fraud does not make it more likely for a man to buy jewellery within 24 hours. Women will behave very differently. Within 24 hours the chance of buying jewellery will increase 10 fold as compared to the case when the fraud has not been committed, and the chance of buying gas will be the same in both cases.
   a) Build a bbn corresponding to the credit- card fraud problem.
   b) What are the independencies and conditional independencies present in the graph?
   c) Specify the conditional probability tables necessary to quantify the model (notice some information is given in the text above but some necessary probabilities are not specified. In such case you should choose your own reasonable numbers. Make it very clear what you specified). Describe clearly your notation.
   d) Calculate the probability that a male committed a credit-card fraud given that he bought today some gas and jewellery.

4) Consider a fault tree with 4 identical basic events A, B, C, D that has the following minimal cut sets {A, B}, {A, C} , {B, D} and {D, C} . Assume that the lifetimes of these basic events are exponential and the binomial failure rate model with parameters lambda = 0.2; mu = 0.1 and p = 0.3 is appropriate to describe common cause failures between these basic events. Calculate the survival function of the top event.

5) **(NonMath)** Let X and Z be independent and exponentially distributed competing risks with parameters λ_X, λ_Z, respectively.

   a) Find the sub-survival function of X and the sub-survival function of Z.
   b) Show that the conditional sub-survival functions of X and Z are equal.
   c) Show that the conditional probability of censoring beyond time t is constant.

5) **(Math)** Assume that experts specify their assessments $x_1, \ldots, x_e$ for the unknown quantity of interests X. Show that when Bayesian technique with additive model is used to combine their opinions with the decision maker prior which is normal with mean $\mu_{e+1} = 0$ and variance σ_{e+1}^2 then the decision maker distribution for X is normal with mean $E(X|x_1 \ldots x_e) = \sum_{i=1}^{e+1} w_i\,(x_i - \mu_i)$ and variance $V(X|x_1 \ldots x_e) = \frac{1}{\sum_{i=1}^{e+1} \sigma_i^{-2}}$ where $w_i = \frac{\sigma_i^{-2}}{\sum_{i=1}^{e+1} \sigma_i^{-2}}$.