1 Since $C(u,v)$ is a bivariate cdf on $[0,1]^2$ then it
a) should be increasing in both arguments, at $(1,1)$ is equal to 1 at $(0,0)$ equal to 0 and the margins are uniform.

It is easy to see that $\alpha \notin (-\infty,-1) \cup (1,\infty)$ otherwise
$C(u,v) \notin (0,1)$

$$c(u,v) = \frac{\partial^2}{\partial u \partial v} C(u,v) = \frac{\partial}{\partial v}\Big[v(1+\alpha(1-u)(1-v)) + uv \cdot \alpha(1-v)(-1)\Big]$$

$$= \frac{\partial}{\partial v}\Big[v + \alpha v(1-u)(1-v) + \alpha uv(1-v)\Big]$$

$$= 1 + \alpha(1-u)(1-2v) - \alpha u(1-2v)$$

$$= 1 + \alpha(1-2v)(1-u-u) = 1 + \alpha(1-2u)(1-2v)$$

Notice if $\alpha \in \langle -1,1 \rangle$ then $c(u,v)$ ~~posi~~ non-negative,
so $C(u,v)$ non-decreasing in both arguments.
We have
$C(1,v) = v$
$C(u,1) = u$
hence it is a copula for $\alpha \in [-1,1]$

b) $$\rho = \frac{E(u,v) - \frac{1}{4}}{\frac{1}{12}} = 12 \int_0^1 \int_0^1 uv(1+\alpha(1-2u)(1-2v))\, du\, dv - 3$$

$$= 12 \int_0^1 v \left[\frac{u^2}{2} + \alpha(1-2v)\left(\frac{u^2}{2} - \frac{2u^3}{3}\right)\right]_0^1 dv - 3$$

$$= 12 \int_0^1 v \left[\frac{1}{2} - \frac{\alpha}{6}(1-2v)\right] dv - 3$$

$$= 12 \left[\frac{v^2}{4} - \frac{\alpha}{6}\left(\frac{v^2}{2} - \frac{2v^3}{3}\right)\right]_0^1 - 3 = 12\left[\frac{1}{4} + \frac{\alpha}{36}\right] - 3 = 3 + \frac{\alpha}{3} - 3$$

$$= \alpha/3$$

c) Let u_1, u_2 be realizations of two independent uniformly distributed random variabels.

$$F_{V|U}(v|u) = \frac{\partial}{\partial u} C(u,v) = v + \alpha v(1-2u)$$ as in a

Then

$$U = u_1$$
$$V = F_{V|U}^{-1}(u_2|u_1).$$

The inverse function of $F_{V|U}$ has to be found numericaly.

(2) Denoting as X - the number of units produced before defective one 3

$P(X=k) = (1-p)^k p \quad , k=0,1,\dots$

a) $L(p \mid data) = P(X=1,p) \cdot P(X=1,p) \cdot P(X=6,p) \cdot P(X=7,p) P(X=2,p)$

$$= (1-p)p\,(1-p)p\,(1-p)^6 p\,(1-p)^7 p\,(1-p)^2 p$$

$$= p^5 (1-p)^{17}.$$

$l = \ln L(p \mid data) = 5 \ln p + 17 (1-p)$

$$\frac{\partial l}{\partial p} = \frac{5}{p} - \frac{17}{1-p} = 0$$

$$5(1-p) = 17p$$

$$5 - 5p = 17p$$

$$\boxed{p = \frac{5}{22}.}$$

b) $p \sim U(0,1)$. Denote as $f_p(p) = 1 \quad 0 \le p \le 1$

$g_{p|}(p) \propto L(p \mid data) \cdot f_p(p)$

$g_p(p) \propto p^5 (1-p)^{17} \cdot 1.$

This is Beta distribution with parameters 6, 18.

c) The prior mean of p is $\frac{1}{2}$ the posterior mean is $\frac{6}{6+18} = \frac{6}{24} = \frac{1}{4}.$

This is not surprising in view of result in a) where $p = \frac{5}{22} = 0.2273$.

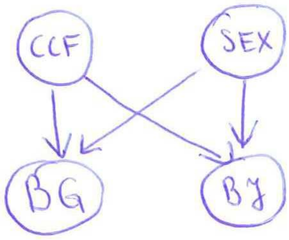
③ a) Nodes :

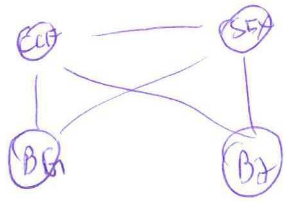
CCF – credit card fraud with states {yes, no}

SEX – {m, f}

BG – buying gas within 24 h. {yes, no}

BJ – buying jewellery -11- {yes, no}





moral graph.

b) $CCF \perp SEX$, $BG \perp BJ \mid CCF, SEX$.

c) We need to specify the following (conditional) probabilities.

$P(CCF=yes) = \underline{0.01}$; $P(SEX=m) = \underline{0.5}$

$P(BG=yes \mid SEX=m, CCF=yes) = 20 \cdot 0.02 = 0.4$

$P(BG=yes \mid SEX=m, CCF=no) = \underline{0.02}$

$P(BG=yes \mid SEX=f, CCF=yes) = 0.03$

$P(BG=yes \mid SEX=f, CCF=no) = \underline{0.03}$

$P(BJ=yes \mid SEX=m, CCF=yes) = \underline{0.001}$

$P(BJ=yes \mid SEX=m, CCF=no) = 0.001$

$P(BJ=yes \mid SEX=f, CCF=yes) = 10 \cdot 0.04 = 0.4$

$P(BJ=yes \mid SEX=f, CCF=no) = \underline{0.04}$

d) $P(CCF=yes \mid SEX=m, BG=yes, BJ=yes) =$

$$= \frac{P(CCF=yes \wedge SEX=m, BG=yes, BJ=yes)}{P(SEX=m, BG=yes, BJ=yes, CCF=yes) + P(SEX=m, BG=yes, BJ=yes, CCF=no)}$$

We get

$P(CCF=yes, SEX=m, BG\ yes, BJ=yes) =$

$P(CCF=yes)\ P(SEX=m) \cdot P(BG=yes \mid CCF=yes, SEX=m)\ P(BJ=yes \mid CCF=yes, SEX=m)$

$= 0.01 \cdot 0.5 \cdot 0.4 \cdot 0.001 = 2 \cdot 10^{-6}$

Similarly

$P(CCF=no, SEX=m, BG=yes, BJ=yes) =$

$= (1-0.01)\ 0.5 \cdot 0.02 \cdot 0.001 = 9.9 \cdot 10^{-6}$

Hence the required conditional probability

$P(CCF=yes \mid SEX=m, BG=yes, BJ=yes) = 0.1681$

④ We have

$$T = AB + AC + BD + DC$$
$$= A(B+C) + D(B+C)$$
$$= (A+D)\cdot(B+C)$$

we need the survival of T, hence T', so we find min paths of this tree.

$$T' = A'\cdot D' + B'\cdot C'$$

we get

$$P(T>t) = P(A>t \wedge D>t \vee B>t \wedge C>t)$$
$$= P(A>t \wedge D>t) + P(B>t \wedge C>t) - P(A>t, B>t, C>t, D>t)$$

Since $\lambda = 0.2$, $\mu = 0.1$, $p = 0.3$ then we get.

$$\lambda_i = \lambda + \mu p(1-p)^3 = 0.2103$$
$$\lambda_{ij} = \mu p^2(1-p)^2 = 0.0044$$
$$\lambda_{ijk} = \mu p^3(1-p) = 0.0019$$
$$\lambda_{ijkl} = \mu p^4 = 0.0008$$

$$P(T>t) = 2\cdot e^{-(2\cdot\lambda_i + 5\lambda_{ij} + 4\lambda_{ijk} + \lambda_{ijkl})t} - e^{-(4\lambda_i + 6\lambda_{ij} + 4\lambda_{ijk} + \lambda_{ijkl})t}$$
$$= 2e^{-0.4510t} - e^{-0.8760t}$$

(5) Math.

Proven on slides 8 - 10 during lectures on Expert Judgement. (Week 10)

(5) Non-Math Solved during the course and proven in the book.

a) We know that

$$\frac{d}{dt} R_z^{*}(t) = R_x(t) \frac{d}{dt} R_z(t)$$

$$R_z^{\alpha}(t) = -\int_t^{\infty} \frac{d}{dt} R_z^{*}(u)\, R_x(u)\, du$$

$$= -\int_t^{\infty} e^{-\lambda_x u} \left(-\lambda_z e^{-\lambda_z u}\right) du$$

$$= \frac{\lambda_z}{\lambda_x + \lambda_z} e^{-(\lambda_x+\lambda_z)t}$$

Similarly for X

$$R_X^{\alpha}(t) = \frac{\lambda_x}{\lambda_x + \lambda_z} e^{-(\lambda_x+\lambda_z)t}$$

b) $$\frac{R_X^{\alpha}(t)}{R_X^{\alpha}(0)} = e^{-(\lambda_x+\lambda_z)t}$$ follows immediately from a)

$$\frac{R_z^{\alpha}(t)}{R_z^{\alpha}(0)} = e^{-(\lambda_x+\lambda_z)t}$$

c) $$\Phi(t) = \frac{R_z^{\alpha}(t)}{R_z^{\alpha}(t) + R_X^{\alpha}(t)} = \frac{\lambda_z}{\lambda_z + \lambda_X}$$ immediately from a)