WI4052 – SOLUTIONS – EXAM RISK – 23.01.2013

①
$$P(X\le s \mid X>t) = \frac{P(X\le s \wedge X>t)}{P(X>t)} = \frac{P(t<X\le s)}{P(X>t)} =$$
$$= \frac{F(s)-F(t)}{1-F(t)} = \frac{1-e^{-\lambda s}-1+e^{-\lambda t}}{e^{-\lambda t}} =$$
$$= \frac{e^{-\lambda t}-e^{-\lambda s}}{e^{-\lambda t}} = \frac{\cancel{e^{-\lambda t}}\left(1-e^{-\lambda(s-t)}\right)}{\cancel{e^{-\lambda t}}}$$
$$= P(X\le s-t)$$

② Prior $p \sim \text{Beta}(5.6,\ 4.4)$ $\quad E(p)=\frac{\alpha}{\alpha+\beta}=\frac{5.6}{5.6+4.4}=0.56$

$$Var(p)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=0.0224.$$

Bernoulli – beta model.

Posterior $\bar{p}$ is $\text{Beta}(\alpha+52,\ \beta+100-52) =$
$\text{Beta}(5.6+52;\ 4.4+48) = \text{Beta}(57.6,\ 52.4)$

$$E(\bar{p}) = \frac{57.6}{57.6+52.4} = 0.5236$$

$$Var(\bar{p}) = 0.0022.$$

(3)

a)

$$\begin{tikzcd}
 & E & \\
VPA \arrow[ur] & & D \arrow[ul] \\
 & Q \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

b) $VPA \perp D \mid Q$, $E \perp Q \mid VPA \wedge D$

c) $P(E) = P(E \mid VPA=l, D)\, P(VPA=l, D) + P(E \mid VPA=m, D)\, P(VPA=m, D)$

$+ P(E \mid VPA=h, D)\, P(VPA=h, D)$

$+ P(E \mid VPA=l, D')\, P(VPA=l, D') + P(E \mid VPA=m, D')\, P(VPA=m,$

$+ P(E \mid VPA=h, D')\, P(VPA=h, D')$

Conditional probabilities are given.
The unconditional probabilities have to be calculated
see e.g.

$P(VPA=l, D) = P(VPA=l, D \mid Q)\, P(Q) + P(VPA=l, D \mid Q')\, P(Q'$

{cond ind} $= P(VPA=l \mid Q)\, P(D \mid Q)\, P(Q) + P(VPA=l \mid Q')\, P(D \mid Q')\, P(Q'$

Hence we get

$P(E) = P(E \mid VPA=l, D)\big[P(VPA=l \mid Q) P(D \mid Q) P(Q) + P(VPA=l \mid Q') P(D \mid Q') P(Q$

$+ P(E \mid VPA=m, D)\big[P(VPA=m \mid Q) P(D \mid Q) P(Q) + P(VPA=m \mid Q') P(D \mid Q') P(Q'$

$+ P(E \mid VPA=h, D)\big[$

$+ P(E \mid VPA=l, D')\big[$

$+$

$+$

etc.

$= 0.01[0.7 \cdot 0.9 \cdot 0.6 + 0.3 \cdot 0.2 \cdot 0.4] + 0.1[0.2 \cdot 0.9 \cdot 0.6 + 0.4 \cdot 0.2 \cdot 0.4]$

....

$= 0.302$

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$M = \min(X, Z)$, hence $M = Y + \min(U, W)$

Moreover, $\min(U,W)$ is exponentially distributed with parameter $\lambda_u + \lambda_w$. Y and $\min(U,W)$ are independent.

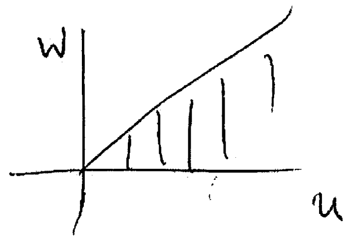
a) $E(M) = E(Y) + E(\min(U,W)) = \frac{1}{\lambda_y} + \frac{1}{\lambda_u + \lambda_w}$

$Var(M) = Var(Y) + Var(\min(W,U)) =$

$$= \frac{1}{\lambda_y^2} + \frac{1}{(\lambda_u + \lambda_w)^2}$$

b) $R_X^*(0) = P(X > 0 \wedge X < Z) = P(X < Z)$

$$= P(Y + W < Y + U) = P(W < U)$$



$$= \int_0^\infty du \int_0^u \lambda_u e^{-\lambda_u u} \cdot \lambda_w e^{-\lambda_w w} \, dw$$

$$= \int_0^\infty \lambda_u e^{-\lambda_u u} \left(1 - e^{-\lambda_w u}\right) du$$

$$= \int_0^\infty \lambda_u e^{-\lambda_u u} - \lambda_u e^{-(\lambda_u + \lambda_w) u} \, du$$

$$= 1 - \frac{\lambda_u}{\lambda_u + \lambda_w}.$$

(5)

0	0.1	0.25	0,5	0,9	1
0	5	10	30	70	100

$$F(x) = \frac{x-0}{100} = \frac{x}{100}$$

- $F(0) = 0$, $F(5) = 0.05$, $F(10) = 0,1$, $F(30) = 0,3$
  $F(70) = 0.7$, $F(100) = 1$

- $r_1 = F(5) - F(0) = 0,05$
  $r_2 = F(10) - F(5) = 0,05$
  $r_3 = F(30) - F(10) = 0,2$
  $r_4 = F(70) - F(30) = 0,4$
  $r_5 = F(100) - F(70) = 0,3$

  $p_1 = 0.1 - 0 = 0.1$
  $p_2 = 0.25 - 0.1 = 0.15$
  $p_3 = 0.5 - 0.25 = 0.25$
  $p_4 = 0.9 - 0.5 = 0,4$
  $p_5 = 1 - 0.9 = 0.1$.

$$I(r) = \sum_{i=1}^{5} p_i \ln(p_i / r_i) = 0.18.$$