Delft University of Technology Mekelweg 4
Faculty EWI

Exam WI 4052TU and WI4052 Solutions, Risk Analysis

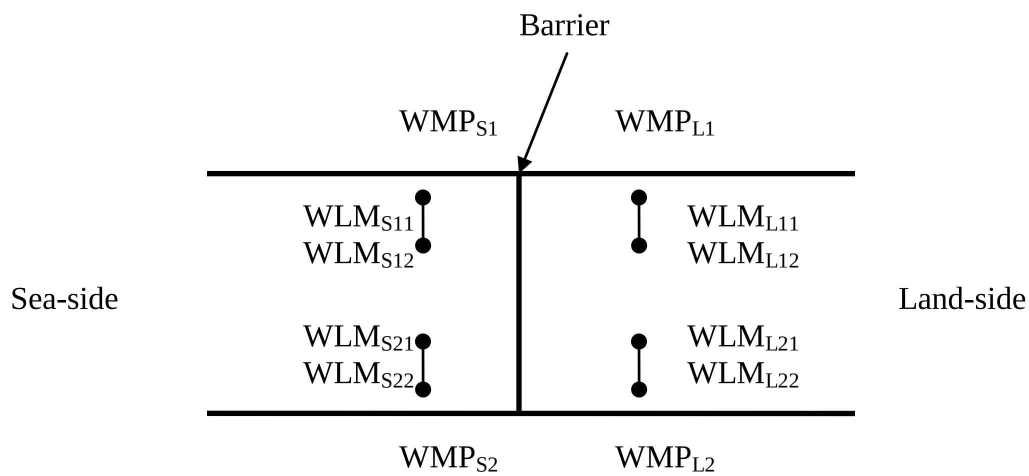
The exam consists of 5 questions. Write your answers in empty spaces between questions. Add a short argumentation for your answers. You are allowed to use a calculator and self made A4 page with formulas. Each exercise is worth 5 points.

1. The component with exponential life time with parameter λ is inspected at regular intervals of length I. If discovered failed during a test, the component is immediately replaced with an identical, independent copy which is as good as new. Given that the average life time of this component is 10 months
   a) what is the optimal length of the inspection interval I such that the availability of this component never drops lower than 0.9 (optimal in this case means the longest, as each inspection costs money and some inconveniency for the production) ?
   We find I such that exp(-1/10*I)=0.9. Hence I=-10*ln(0.9)= 1.0536(a bit more than 1 month).
   b) For the optimal I obtained in a) find the equilibrium availability of this component and explain what this number means.
   The equilibrium availability is the availability averaged over infinite horizon and is in this case approximately equal 1-(1/10*1.0536)/2=0.9473.

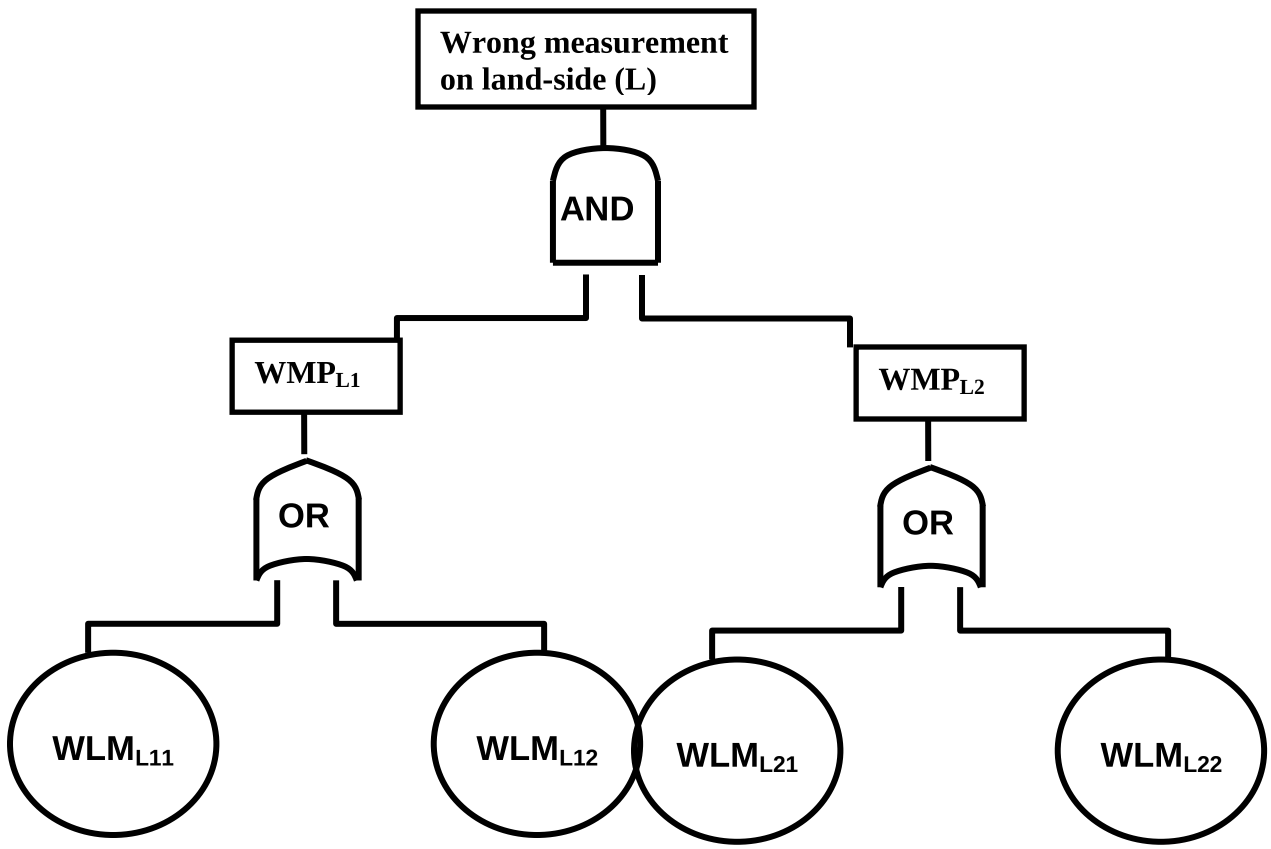
2. The Storm Surge Barrier in the New Waterway near Rotterdam is designed to protect the local population against high water. Because of the choice of design (two floating doors mounted on a ball hinge), the barrier is not able to withstand high pressure from the river. If the river water level is more than one meter higher than the sea level the barrier will fail.

   A water measurement system has been installed to measure the water levels on both sides of the barrier, so that the barrier can be opened before the water-level difference is too high. The barrier is equipped with four water-level measurement posts (WMP), each of which contains two water-level meters (WLM). There is a water-level measurement post by each bank, on both the see and the land side.

   We assume that correct measurement from one WMP on each side of the barrier is sufficient to determine on time whether the barrier should be opened. To simplify the story we assume that the correct measurement of a WMP is obtained if both WLMs work. Moreover we assume that WLMs fail independently with probability 0.001. What is the probability of failure if the Storm Surge Barrier in the New Waterway near Rotterdam?



Barrier
WMP$_{S1}$
WMP$_{L1}$
WLM$_{S11}$
WLM$_{S12}$
WLM$_{L11}$
WLM$_{L12}$
Sea-side
Land-side
WLM$_{S21}$
WLM$_{S22}$
WLM$_{L21}$
WLM$_{L22}$
WMP$_{S2}$
WMP$_{L2}$



Wrong measurement on land-side (L)
AND
WMP$_{L1}$
WMP$_{L2}$
OR
OR
WLM$_{L11}$
WLM$_{L12}$
WLM$_{L21}$
WLM$_{L22}$

The part of FT for wrong measurement on the land side (L) is shown on the Figure above. Similar part of FT for the wrong measurement on the seas side (S) should be drawn. These two intermediate events should be connected by OR gate. Hence the probability of failure of the Storm Surge Barrier in the New Waterway near Rotterdam (FB) is:

$P(FB) = P(L \text{ or } S) = P(WMP_{L1} \text{and } WMP_{L2} \text{ or } WMP_{S1} \text{and } WMP_{S2}) =$
$= P(WMP_{L1} \text{and } WMP_{L2}) + P(WMP_{S1} \text{and } WMP_{S2}) - P(WMP_{L1} \text{and } WMP_{L2} \text{ and } WMP_{S1} \text{and } WMP_{S2}) =$
$= P((WLM_{L11} \text{or } WLM_{L12}) \text{and } (WLM_{L21} \text{or } WLM_{L22}))$
$+ P((WLM_{S11} \text{or } WLM_{S12}) \text{and } (WLM_{S21} \text{or } WLM_{S22}))$
$- P((WLM_{L11} \text{or } WLM_{L12}) \text{and}(WLM_{L21} \text{or } WLM_{L22}) \text{and}(WLM_{S11} \text{or } WLM_{S12}) \text{and}(WLM_{S21} \text{or } WLM_{S222}))$

All WLMs fail independently with probability p=0.001 so we get:
$P(FB) = 2*(2p-p^2)^2-(2p-p^2)^4 = p^2(2-p)^2(2-p^2(2-p)^2) = 7.9920e-006$

3. Consider a system of three identical components which fails when any two components fail. We assume that the components have an exponential life distribution and that the binomial failure rate model can be used to model common cause failures. Over past 100 operating months of this system, we have observed the following: 20 single component failures, 10 failures of two components simultaneously, 5 failures of three components simultaneously.

a. What are the estimates for parameters of this model?

$T=100$, $n_1=20$, $n_2=10$, $n_3=5$.
$n_+ = n_2 + n_3 = 10+5=15$,
$S=2n_2+3n_3=2*10+3*5=35$
Since $S=n_+mp(1-q^{m-1})/(1-q^m-mpq^{m-1})$ then we must solve for p and q using the following equation
$35=15*3*p(1-(1-p)^2)/(1-(1-p)^3-3p(1-p)^2)$
For 3 components we get $p=3*(35-2*15)/(2*35-3*15)=0.6$
$\lambda_1=n_1/T=0.2$,
$\lambda_+ = n_+/T=0.15$
$\lambda_+=\mu(1-q^m-mpq^{m-1})$ then $\mu= 0.15/(1-(1-0.6)^3-3*0.6*(1-0.6)^2)= 0.1603$
$\lambda_1= m\lambda+m\mu\lambda pq^{m-1}$ then $\lambda= 0.2/[3*(1+0.6*(1-0.6)^2)]= 0.0608$

b. What is the reliability of the system for a mission time of 12 months?

(*) $Pr(S>t)=Pr(T_1>t_1, T_2>t_2)+ Pr(T_1>t_1, T_3>t_3) + Pr(T_2>t_2, T_3>t_3)$
$-2 Pr(T_1>t_1, T_2>t_2, T_3>t_3)$

$Pr(T_1>t_1, T_2>t_2, T_3>t_3)$
$=\exp(-\lambda_1 t_1-\lambda_2 t_2-\lambda_3 t_3-\lambda_{21} \max\{t_1, t_2\}-\lambda_{31} \max\{t_1, t_3\}-\lambda_{23} \max\{t_2, t_3\}-\lambda_{123} \max\{t_1, t_2, t_3\})$
$Pr(T_1>t, T_2>t, T_3>t)$
$=\exp([-\lambda_1-\lambda_2-\lambda_3-\lambda_{21}-\lambda_{31}-\lambda_{23}-\lambda_{123}]t)$

Margins can be also found.
$Pr(T_1>t)$
$=\exp([-\lambda_1 -\lambda_{21}-\lambda_{31}-\lambda_{123}]t)$

$Pr(T_1>t, T_2>t)$
$=\exp([-\lambda_1-\lambda_2 -\lambda_{21}-\lambda_{31}-\lambda_{23} -\lambda_{123}]t)$

We get that λ's are:

$\lambda_i= \lambda+p(1-p)^2 \mu$ =0.0608+0.6*(1-0.6)^2*0.1603=0.0762
$\lambda_{ij}= p^2(1-p)\mu$=0.6^2*(1-0.6)*0.1603=0.0231
$\lambda_{ijk}= p^3\mu$=0.6^2*0.1603=0.0577

From the above and the formula (*) we get
$Pr(S>t)=3*e^{-0.2793*t} -2*e^{-0.3555*t}$
Pr(S>12)= 0.0770

4. Two experts provide their assessments of the average weight of an adult in China. The estimate of the first expert is e_1=60 and the second one is e_2=70. The analyst considers both experts equally reliable and assigns to each an additive standard normal error. As the prior distribution of weight the analyst takes the world wide distribution of weight which is normal with mean 75 and standard deviation 20. What is the Bayesian combination of experts' assessments?
Using formulas from proposition 10.1 on page 193 we get
$E(x|x_1,x_2)=w_1*(60-0)+w_2(70-0)+w_3(75-0)$= 65.0125
where
w_1=1/(1+1+1/20^2)= 0.4994
w_2=1/(1+1+1/20^2)= 0.4994
w_3=(1/20^2) /(1+1+1/20^2)= 0.0012
$V(x| x_1,x_2)$= 1/(1+1+1/20^2)= 0.4994.
Hence combination of expert assessments is normal with mean 65.0125 and variance 0.4994.
5. The following data contains times (in operating hours) of occurrences of bugs in a software:
1, 3, 6, 10, 15
t_i: 1, 2, 3, 4, 5
T=15
   a. State the assumption of the Jalinski-Moranda model and check that the parameters of this model for this data are N≈5.4468 and φ ≈0.1199.
   See the book or lecture notes for assumptions.
   Use the given values for N and φ and see that they satisfy the likelihood equations presented during the course and given in the book on page 248.
   b. What is the distribution of time to the occurrence of the next bug?
   It is exponential with parameter (N-6)*φ.
   c. Check if this model underestimates or overestimates the data.

   $F=1-\exp(-(5.4468-(i-1))*0.1199t_i)$

[0.4796 0.6557 0.7106 0.6907 0.5799]

After sorting these values we get

[0.4796 0.5799 0.6557 0.6907 0.7106]

which should be realizations of uniform distribution. We plot these numbers against the quantile values and see that most of the time they are above the diagonal so the model is overestimated. However there were only 5 points so it is not easy to draw this conclusion.