Exam Signal Processing TI2710-A

January 22$^{\text{th}}$, 2013

Question 1 (6 points total)

A signal processing student, who is a singer in a band in his spare time, decides to bring his knowledge into practice by processing his own voice. To do so, he records his voice with a microphone. The microphone is attached to an analog-to-digital converter, which is connected to a computer with some signal processing software installed on it (e.g., Matlab). The human voice can produce sounds from approximately 80 Hz, up to 1100 Hz.

(a) (1 p.) Choose a sampling frequency f_s that is used to sample the recorded signal in the analog-to-digital converter. Motivate your answer (there is no unique answer).

To test the system, the student first creates a synthetic signal that is recorded by the microphone. The recorded signal is given by

$$x(t) = \cos(2\pi 50t) + \cos(2\pi 800t).$$

(b) (2 p.) For the sampling frequency chosen in (a), give the expression for the sampled time-discrete signal $x[n]$.

Using an implementation of the Fourier transform, the student wants to analyze the recorded signal.

(c) (1 p.) Which version of the Fourier transform is used by the student when he uses his computer to analyze the signal. Motivate your answer.

The student notices that the recording contains the component of 50 Hz. This is a distortion introduced by the power supply. To remove this distortion, he wishes to design a filter that removes this component, while preserving frequencies that will be produced by the human voice.

(d) (1 p.) Sketch the magnitude response of a discrete-time filter ($|H(e^{j\hat{\omega}}|$) that removes the component of 50 Hz, but preserves any sounds produced by the human voice.

The signal is filtered, leading to a discrete-time signal $y[n]$ where the 50 Hz component is removed completely. The student plays the signal $y[n]$ using a loudspeaker attached to the computer and a build-in digital-to-analog converter.

(e) (1 p.) Argue which interpolation method could be used and plot 2 periods of $y[n]$ and $y(t)$.

Question 2 - Linear time-invariant (LTI) systems (8 points total)

A discrete-time system is described by the following input-output relation:

$$y[n] = x[n] + nx[n-1].$$

(a) (2 p.) Show whether or not this system is linear.

Given are two LTI systems S_1 and S_2 with the following impulse responses:

$$S_1: \quad h_1[n] = 2\delta[n] - \delta[n-1],$$

and

$$S_2: \quad h_2[n] = 2\delta[n] + 2\delta[n-1].$$

These two LTI systems are cascaded as illustrated in Figure 1 on page 4.

(b) (1 p.) Explain why the order in which systems S_1 and S_2 are cascaded does not influence the impulse response of the cascaded system S_3.

(c) (2 p.) Show that the impulse response of the cascaded system S_3 is given by

$$S_3: \quad h_3[n] = 4\delta[n] + 2\delta[n-1] - 2\delta[n-2].$$

(d) (1 p.) The input to the cascaded system S_3 is given by $x[n] = \delta[n] + 2\delta[n-2]$. Compute the output $y[n]$ of system S_3.

Now consider the systems S_4 and S_5 with the following input-output relations:

$$S_4: \quad y_4[n] = nx_4[n-1] + x_4[n],$$

and

$$S_5: \quad y_5[n] = x_5[n-1].$$

These two systems are also cascaded. System S_4 is not an LTI system, as it is time-variant.

(e) (2 p.) Show, using an example, that in this case the order in which the two systems are cascaded matters.

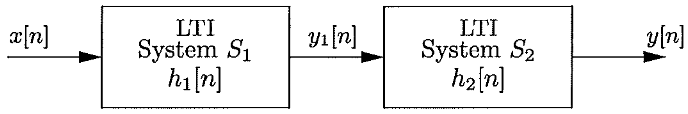

Figure 1: Cascaded system S_3, composed of the LTI systems S_1 and S_2.

Question 3 - Transfer functions (9 points total)

The impulse response of an FIR system S_1 is given by

$$h_1[n] = 2\delta[n] + 2\delta[n-2].$$

(a) (2 p.) Show that the transfer function of the FIR system equals:

$$H_1(e^{j\hat{\omega}}) = 4e^{-j\hat{\omega}}\cos(\hat{\omega}).$$

(b) (1 p.) Sketch the magnitude response of $H_1(e^{j\hat{\omega}})$ for $0 \leq \hat{\omega} \leq 4\pi$. Clearly mark the axes and indicate zeroes and maxima in the sketch.

(c) (1 p.) Sketch the principal value of the phase response of $H_1(e^{j\hat{\omega}})$ for $0 \leq \hat{\omega} \leq 4\pi$.

Given is a second FIR system S_2 with input-output relation

$$y_2[n] = 2x_2[n-1]$$

(d) (2 p.) System S_1 and S_2 are put in cascade. Determine the transfer function $H_3(e^{j\hat{\omega}})$ of the cascaded system.

Consider the following input signal:

$$x[n] = \cos\left(\frac{\pi}{2}n + \frac{\pi}{2}\right) + 2\cos\left(\pi n\right).$$

(e) (1 p.) Plot the complex spectrum of $x[n]$. Clearly indicate the complex amplitudes $X(e^{j\hat{\omega}})$.

(f) (2 p.) Determine the output $y[n]$ of the cascaded system S_3 for input signal $x[n]$ using the result obtained in Question (d).

Question 4 - Discrete Fourier transformations (DFTs)(6 points total)

A discrete-time signal $x[n]$ is given by

$$x[n] = \begin{cases} 2 & n = 0, \\ 2 & n = 4, \\ 0 & n = 1,\ 2,\ 3,\ 5,\ 6,\ 7. \end{cases}$$

The N-point DFT is given by

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}.$$

(a) (2 p.) Compute the 8-point DFT of this signal and show that it equals

$$X[k] = 4e^{-j\pi k/2} \cos\left(\frac{\pi k}{2}\right) \quad \text{for } k = 0,\ 1, ..., 7.$$

(b) (1 p.) Give the expression for the magnitude of $X[k]$, and make a plot of $|X[k]|$.

The signal $x[n]$ is input to an LTI filter with impulse response

$$h[n] = \begin{cases} \frac{1}{2} & n = 6, \\ 0 & n = 0,\ 1, ..., 7. \end{cases}$$

(c) (1 p.) Compute the transfer function $H[k]$ for $N = 8$ of this impulse response.

(d) (1 p.) Compute the DFT $Y[k]$ of output $y[n]$.

(e) (1 p.) Plot the magnitude and phase spectrum of output $y[n]$.

Question 5 - Fourier series (10 points total)

Given is the periodic signal $x(t)$,

$$x(t) = 5 + 5\cos(2\pi(20)t) + 10\cos(2\pi(30)t + \pi/3).$$

(a) (1 p.) Determine the fundamental frequency f_0 and the fundamental period T_0.

The Fourier synthesis expression is given by

$$x(t) = \sum_{k=-N}^{N} a_k e^{j\frac{2\pi}{T_0}kt}.$$

(b) (2 p.) Determine all the complex amplitudes a_k for the signal $x(t)$. (*Hint: it is not necessary to compute any integrals for this.*)

(c) (1 p.) Plot the complex spectrum of $x(t)$ using the coefficients computed in question (b). Clearly mark the scale of the axes of the complex spectrum.

Let another time-continuous periodic signal with period $T_0 = 2$ be described over one period by

$$z(t) = \begin{cases} 0 & 0 \le t < 1 \\ 2 & 1 \le t < 2. \end{cases}$$

The Fourier series analysis integral (continuous time, discrete frequency) is defined as

$$a_k = \frac{1}{T_0}\int_0^{T_0} z(t)e^{-j(2\pi/T_0)kt}dt.$$

(d) (2 p.) Compute the Fourier coefficient a_0 of the signal $z(t)$.

(e) (2 p.) Show that the Fourier coefficients a_1 and a_2 of the signal $z(t)$ are given by $a_1 = \frac{-2}{j2\pi}$ and $a_2 = 0$.

(f) (1 p.) Find the Fourier coefficients a_{-1} and a_{-2}.

(g) (1 p.) Plot the magnitude spectrum and the phase spectrum of $z(t)$ based on the Fourier coefficients a_{-2}, a_{-1}, a_0, a_1 and a_2. Clearly mark the scale of the axes of the complex spectrum.