Examination **Random Signal Processing** (IN4309)

Part I: Digital Signal Processing

November 8, 2012
(14:00 - 17:00)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1:

Determine the $\mathcal{Z}$-transform of the following signals, sketch the corresponding pole-zero plot, and indicate what the region of convergence is.

a)

$$x(n) = a^{n-1}u(n-1)$$

b)

$$x(n) = (1+n)u(n)$$

c)

$$x(n) = \begin{cases} 1, & n = 0, \ldots, 10 \\ 0, & \text{otherwise} \end{cases}$$

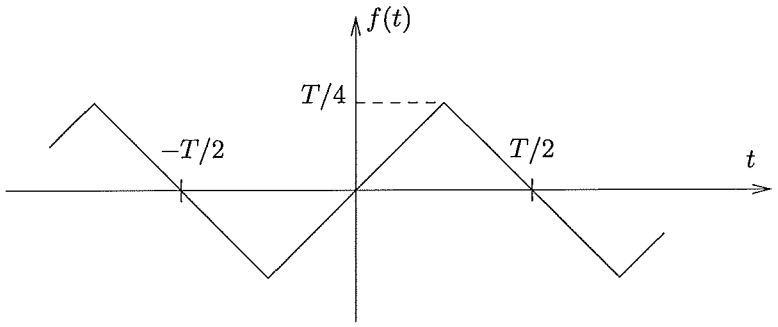

Figure 1:

Assignment 2:

Consider the periodic function f as depicted in Figure 1. A Fourier series representation of f is given by

$$f(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos\left(\frac{2\pi k}{T}t\right) + b_k \sin\left(\frac{2\pi k}{T}t\right),$$

where

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt,$$

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi k}{T}t\right)dt,$$

and

$$b_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi k}{T}t\right)dt.$$

a) Without explicitly computing the coefficients a_k and b_k, what can you say about the values of a_k and b_k? What is the order of decay of the Fourier coefficients? Motivate your answer.

b) Compute the a_k and b_k.

Assignment 3:

Consider a speech signal s given by

$$s(t) = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t), \tag{1}$$

consisting of two harmonics where $f_1 = 1$ kHz and $f_2 = 5$ kHz.

a) Is this signal periodic? If so, what is the fundamental frequency?

b) Give the Fourier transform of the signal.

Suppose we sample the speech signal s with sampling frequency f_s. Afterwards we can reconstruct the (analog) speech signal out of its samples, say $s(n)$, by a proper interpolation scheme.

c) How does the ideal interpolation scheme look like and motivate why it is ideal. What is the reconstruction formula?

d) What is the minimum sampling frequency needed for error-free reconstruction in the case we use an ideal interpolation scheme? Motivate why this is the correct solution.

e) Assume $f_s = 8$ kHz. Give an expression of the reconstructed speech signal, say $\tilde{s}(t)$. Is the reconstruction perfect?

f) Given a sampling frequency of $f_s = 16$ kHz, do there exist other, non-ideal, interpolation scheme that can be used to perfectly reconstruct s as given by (1) out of its samples $s(n)$?

Assignment 4:

Consider a causal linear time-invariant system, having zeros at $z = 0$ and $z = \frac{1}{2}a$ and poles at $z = ae^{j\frac{\pi}{3}}$ and $z = ae^{-j\frac{\pi}{3}}$.

a) Give the corresponding pole-zero plot.

b) Determine the corresponding system function. What is the region of convergfence? Is this function unique?

c) Is this a BIBO stable system? Motivate your answer.

d) Determine and sketch the magnitude response of the system.

e) Compute the inverse $\mathcal{Z}$-transform of the system function $H(z)$ in case the region of convergence is $|z| > a$.

f) Compute the impulse response of the system. What is the steady-state response and what is the transient response?

g) Compute the step response of the system. What is the steady-state response and what is the transient response?