Assignment 1:

Consider the following transfer function:

$$H(z) = \frac{-z}{z^2 - 2z + \frac{3}{4}}.$$

a) Determine the poles and zeros of $H(z)$, and plot the pole-zero map

Consider the following regions of convergence:

R_1: $|z| > \frac{3}{2}$

R_2: $|z| < \frac{1}{2}$

R_3: $\frac{1}{2} < |z| < \frac{3}{2}$

b) Compute the inverse $\mathcal{Z}$-transform of $H(z)$ when the region of convergence is R_1.

c) Compute the inverse $\mathcal{Z}$-transform of $H(z)$ when the region of convergence is R_2.

d) Compute the inverse $\mathcal{Z}$-transform of $H(z)$ when the region of convergence is R_3.

Assignment 2:

A discrete-time signal x has the following Fourier transform

$$X(\omega) = \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1.$$

a) Give an expression for $x(n)$.

Give the Fourier transforms of the following signals

b) $x(2n+1)$

c) $(x * x)(n)$

d) $x(n)\cos(\frac{\pi}{3}n)$

Assignment 3:

Consider the continuous-time signal

$$x_a(t) = \begin{cases} e^{-\alpha t} e^{-j2\pi f_0 t}, & t \geq 0 \\ 0, & t < 0 \end{cases}, \quad \alpha > 0.$$

a) Show that

$$X_a(f) = \frac{1}{j2\pi(f + f_0) + \alpha}.$$

b) Sketch the magnitude spectrum of X_a for $f_0 = 10$ Hz

The signal x_a is sampled to obtain the discrete-time signal x.

c) Sketch the magnitude spectrum of $X(f)$, the Fourier transform of the discrete-time signal x, for $f_s = 10, 20$ and 40 Hz and explain the results in terms of aliasing effects.

d) What is the minimum sampling frequency f_s such that the continuous-time signal x_a can be perfectly recovered from its samples $x(n)$?

Assignment 4:

Consider the transfer function

$$H(z) = \frac{z^2 - 1}{z^2 + \frac{1}{4}},$$

of a causal linear time-invariant system with its direct form-II implementation shown below.

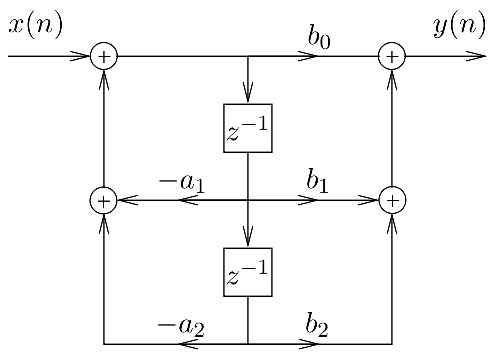

Figure 1: Direct form II implementation of $H(z)$

a) What are the values of the filter coefficients b_0, b_1, b_2, a_1 and a_2?

b) What can you say about the pole positions and stability of the system? (*Hint:* use the stability triangle)

c) What are the zeros and poles of $H(z)$ and plot the pole-zero map

d) Sketch the corresponding magnitude and phase response

Consider the following (suddenly applied) input signal

$$x(n) = \left(e^{j\frac{\pi}{2}n} + e^{j\pi n}\right) u(n),$$

and assume the system is initially in rest.

e) Compute the steady-state response $y_{\text{ss}}(n)$ when $x(n)$ is input to the system

f) Compute the total response of the system $y(n) = y_{\text{tr}}(n) + y_{\text{ss}}(n)$, where $y_{\text{tr}}(n)$ denotes the transient response of the system