Examination **Random Signal Processing** (IN4309)

Part I: Digital Signal Processing

November 11, 2011
(14:00 - 17:00)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1:

Given $X(z)$, the $\mathcal{Z}$-transform of $x(n)$. Give the inverse $\mathcal{Z}$-transform of

a)

$$z^k X(z)$$

b)

$$X(-z)$$

c)

$$X(z^{-1})$$

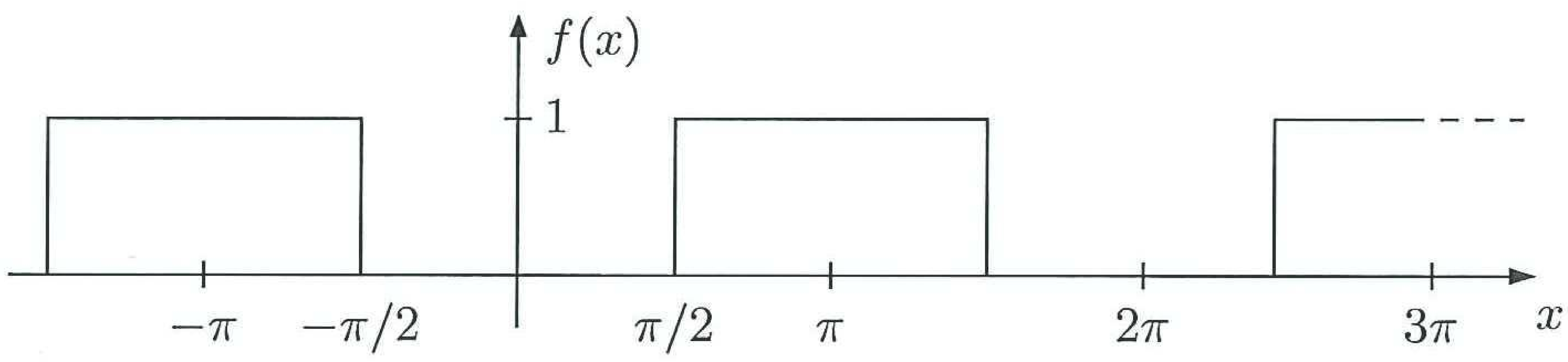

Figure 1: Periodic function.

Assignment 2:

Consider the periodic function f as depicted in Figure 1. A Fourier series representation of f is given by

$$f(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

a) Without explicitly computing the coefficients a_k and b_k, what can you say about the values of a_k and b_k? What is the order of decay of the Fourier coefficients? Motivate your answer.

b) Compute the a_k and b_k.

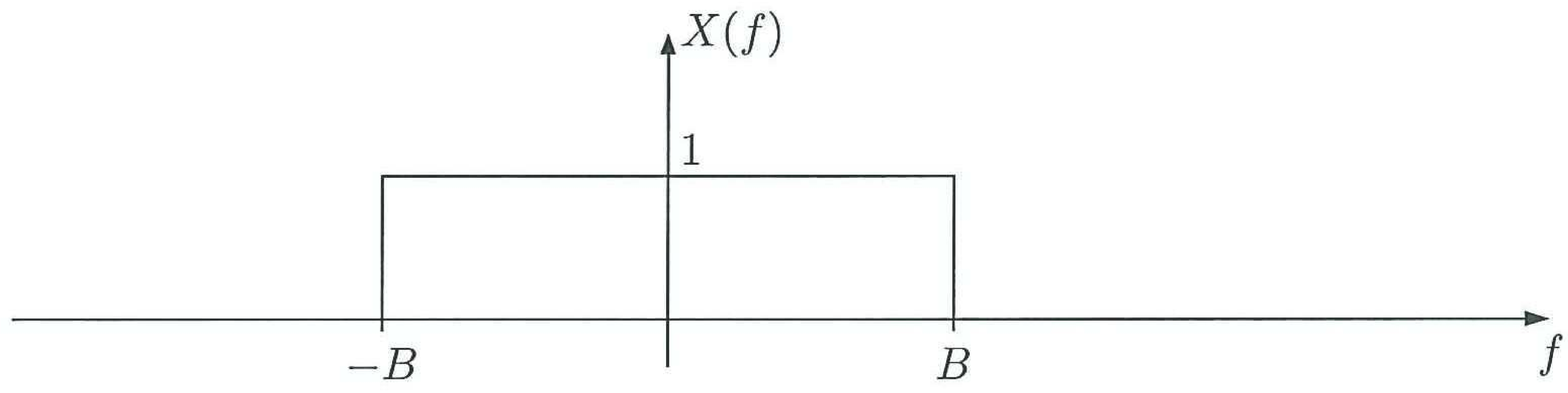

Figure 2: Spectrum $X(f)$.

Assignment 3:

Consider a signal x of which its spectrum is given as depicted in Figure 2.

a) Is this signal x a discrete-time or continuous-time signal? Motivate your answer.

b) Is the signal x a periodic or a non-periodic signal? Motivate your answer.

Suppose we sample the signal $x(t)$ with sampling frequency $f_s = 3B$. Afterwards we can reconstruct the (analog) signal out of its samples, say $x(n)$, by a proper interpolation scheme.

c) What is the relation between $x(t)$ and $x(n)$?

d) Sketch the spectrum of $x(n)$.

e) What is the minimum sampling frequency such that we can perfectly reconstruct $x(t)$ out of its samples $x(n)$ and give the corresponding reconstruction formula.

Assume we want to reconstruct the continuous-time signal using the interpolation function as depicted in Figure 3.

f) How does the continuous-time reconstructed signal look like if we reconstruct using the above mentioned interpolation function?

The reconstruction formula can be expressed in the Fourier domain and is given by

$$\tilde{X}(f) = X(f)G(f),$$

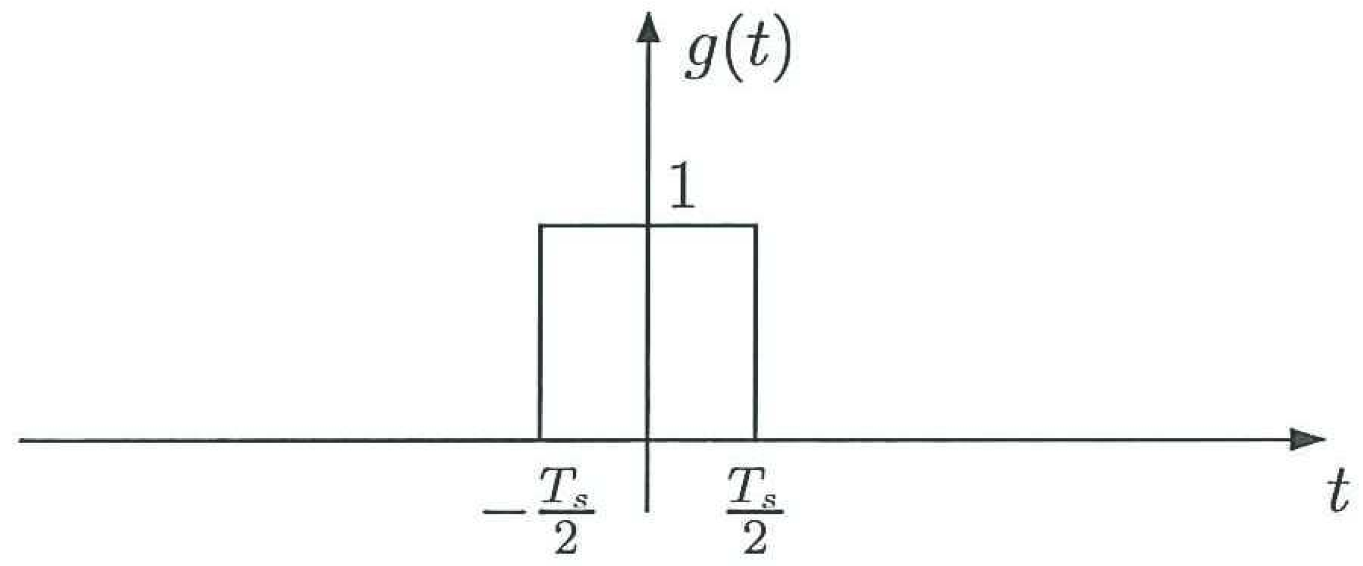

Figure 3: Interpolation function.

where $\tilde{X}(f)$ is the reconstructed signal, $X(f)$ is the spectrum of the discrete-time signal $x(n)$ and $G(f)$ is the Fourier transform of the interpolation function.

g) Compute $G(f)$.

h) Sketch the spectrum of $\tilde{X}(f)$

i) Can we obtain a perfect reconstruction of $x(t)$ by using the interpolation function depicted in Figure 3? Motivate your answer.

Assignment 4:

Consider the following pole-zero map of a causal linear time-invariant system, having zeros at $z = 0$ and $z = r\cos(\theta)$ and poles at $z = re^{j\theta}$ and $z = e^{-j\theta}$.

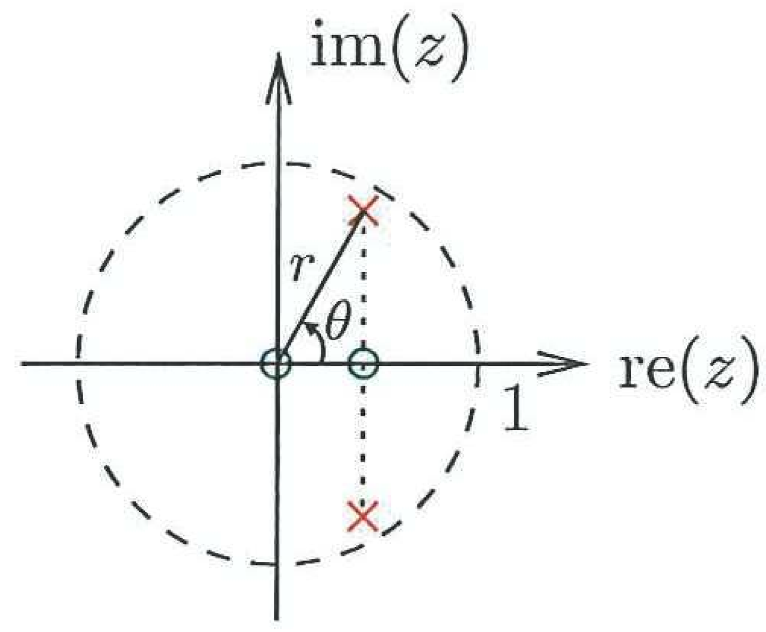

Figure 4: Pole-zero map.

a) Determine the corresponding system function. What is the region of convergence? Is this function unique?

b) What are the conditions for r and θ in order to have a BIBO stable system? Motivate your answer.

c) Determine and sketch the magnitude response of the system.

d) Compute the inverse $\mathcal{Z}$-transform of the system function $H(z)$ in case the region of convergence is $|z| > r$.

e) Assume the impulse response of the system is given by $h(n) = r^n \cos(n\theta)$. Just by looking to the impulse response itself, what can you say about the conditions for r and θ in order to have a BIBO stable system? How do these results relate to the ones obtained under item b)?

Consider the following (suddenly applied) input signal

$$x(n) = e^{j\frac{\pi}{4}n}u(n),$$

and assume the system is initially in rest.

f) Compute the total response of the system $y(n) = y_{\text{tr}}(n) + y_{\text{ss}}(n)$, where $y_{\text{tr}}(n)$ and $y_{\text{ss}}(n)$ denote the *transient* and *steady-state* response of the system, respectively.