Examination **Random Signal Processing** (IN4309)

Part I: Digital Signal Processing

January 21, 2011
(13:00 - 16:00)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1:

Consider the following pole-zero map of a causal linear time-invariant system.

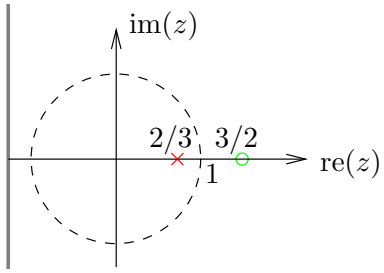

Figure 1: Pole-zero map.

a) Determine the corresponding system function. Is this function unique?

b) What is the region of convergence?

c) Is this system BIBO stable? Motivate your answer.

d) Determine and sketch the magnitude response of the system.

e) Compute the inverse $\mathcal{Z}$-transform of the system function $H(z)$.

Assignment 2:

Compute the $\mathcal{Z}$-transform of the following signals:

a)

$$h(n) = (1+n)u(n).$$

b)

$$h(n) = (-1)^n a^n.$$

c)

$$h(n) = (-1)^n a^{-n}.$$

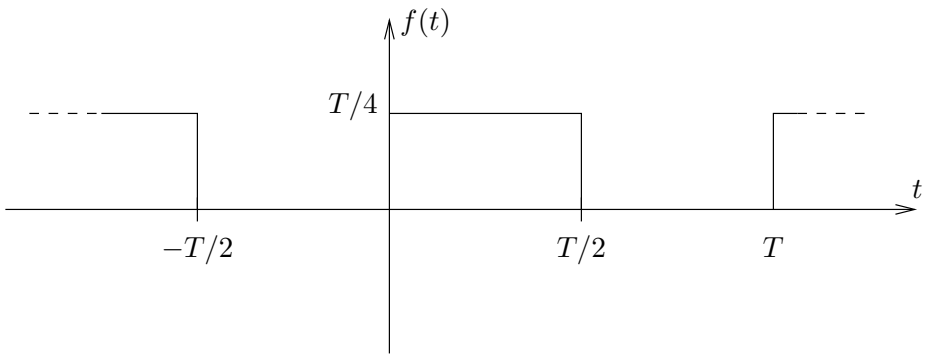

Figure 2: Periodic function.

Assignment 3:

Consider the periodic function f as depicted in Figure 2. A Fourier series representation of f is given by

$$f(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos\left(\frac{2\pi k}{T}t\right) + b_k \sin\left(\frac{2\pi k}{T}t\right),$$

where

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt,$$

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi k}{T}t\right)dt,$$

and

$$b_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi k}{T}t\right)dt.$$

a) Without explicitly computing the coefficients a_k and b_k, what can you say about the values of a_k and b_k? What is the order of decay of the Fourier coefficients? Motivate your answer.

b) Compute the a_k and b_k.

Assignment 4:

Consider the following realization of a discrete-time system for which a_1, a_2 and b_0 are real.

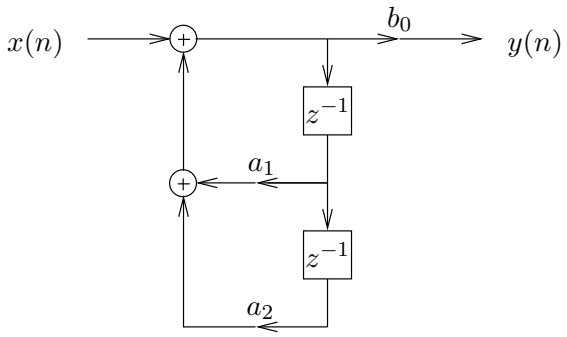

Figure 3: Block diagram realization.

a) Show that this system is linear.

b) Show that the system in time invariant.

c) Show that the input-output difference relation is given by

$$y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 x(n).$$

d) Give the corresponding transfer function.

e) What are the poles and what are the zeros of this system?

Let

$$a_1 = -\frac{1}{2} \text{ and } a_2 = \frac{1}{2}.$$

and assume the system is initially in rest.

f) Compute and sketch the impulse response of the system.

g) Compute and sketch the step response of the system.

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and assume the system is initially in rest.

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