Examination **Random Signal Processing** (IN4309)

Part I: Digital Signal Processing

November 6, 2009
(14:00 - 17:00)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1:

Consider the following pole-zero map of a causal linear time-invariant system.

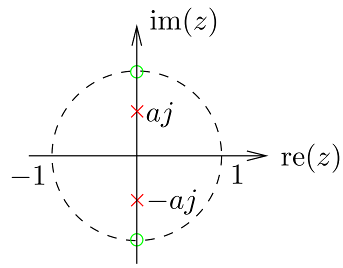

Figure 1: Pole-zero map.

a) What is the region of convergence (ROC)? Motivate your answer.

b) Determine the corresponding system function, say $H(z)$.

c) Is this system BIBO stable? Motivate your answer.

d) Compute the inverse $\mathcal{Z}$-transform of $H(z)$.

We can scale the transfer function found in b) without changing its corresponding pole-zero map. That is, the function $G(z) = cH(z)$ has the same poles and zeros.

e) What is the scaling c we have to apply to obtain $g(0) = 1$?

f) Determine and sketch the magnitude response of the system.

Assignment 2:

Consider the continuous-time signal $x_a(t)$ whose magnitude and phase spectrum are shown in Figure 2a) and b), respectively.

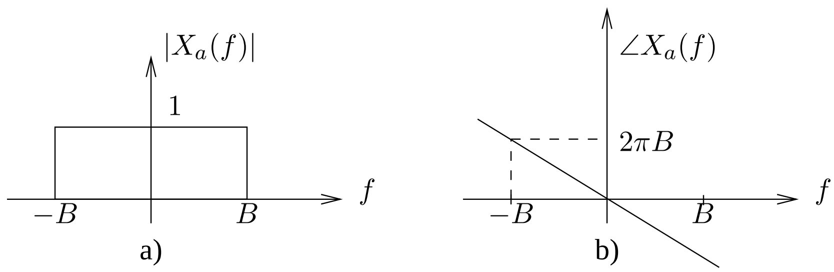

Figure 2: Magnitude a) and phase b) spectrum.

a) Show that

$$x_a(t) = \frac{\sin(2\pi B(t-1))}{\pi(t-1)}.$$

Suppose we want to sample the signal x_a to obtain the discrete-time signal x given by

$$x(n) = x_a(nT_s),$$

where T_s denotes the sampling period.

b) What is the minimum sampling frequency f_s such that the continuous-time signal $x_a(t)$ can be perfectly recovered from its samples $x(n)$?

c) Determine the Fourier transform $X(f)$ of $x(n)$ and sketch its magnitude spectrum.

Consider a particular harmonic component of x_a, say h_a, given by

$$h_a(t) = 2\cos(2\pi f_0(t-1)),$$

where $B/2 < f_0 < B$, and assume this harmonic is sampled at a sampling frequency $f_s = B$.

d) What will be the frequency of the reconstructed harmonic if we use an ideal interpolation scheme?

e) Is it possible to sample the spectrum $X(f)$ and to reconstruct $x(n)$ out of these spectral samples? Motivate your answer.

Assignment 3:

Consider the FIR filter

$$y(n) = x(n) + x(n-3).$$

a) Determine the transfer function and draw the pole-zero map

b) Compute and sketch the magnitude and phase response

Consider the following input signal

$$x(n) = \delta(n) + e^{j\frac{\pi}{3}n}u(n) + e^{j\frac{2\pi}{3}n}u(n).$$

c) Compute the steady-state response $y_{\mathrm{ss}}(n)$ when $x(n)$ is input to the system

d) Compute the total response $y(n) = y_{\mathrm{ss}}(n) + y_{\mathrm{tr}}(n)$, where $y_{\mathrm{tr}}(n)$ denotes the transient response.