Examination **Random Signal Processing** (IN4309)

October 28, 2008
(14:00 - 17:00)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1:

Consider the following pole-zero map.

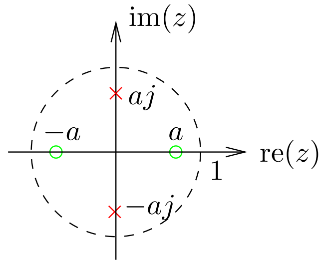

Figure 1: Pole-zero map.

a) Give the corresponding transfer function $H(z)$. Is this function unique given the pole-zero map shown above?

Consider the following regions of convergence:

R_1: $|z| > a$

R_2: $|z| < a$

b) Just by looking to the pole-zero map, what can you say about the stability of the system for the two different regions of convergence? And what about the causality of the system?

c) Compute the inverse $\mathcal{Z}$-transform of $H(z)$ when the region of convergence is R_1.

d) Compute the inverse $\mathcal{Z}$-transform of $H(z)$ when the region of convergence is R_2.

Assignment 2:

Consider the continuous-time signal

$$x_a(t) = \begin{cases} \alpha^t, & t \geq 0 \\ 0, & t < 0 \end{cases}, \quad \alpha < 1.$$

a) Show that

$$X_a(f) = \frac{1}{j2\pi f - \ln \alpha}.$$

b) Sketch the magnitude spectrum of X_a.

The signal x_a is sampled to obtain the discrete-time signal x given by

$$x(n) = \beta^n u(n).$$

c) What is the value of β?

d) Compute the Fourier transform $X(f)$ of $x(n)$.

e) Sketch the magnitude spectrum of $X(f)$.

f) What is the minimum sampling frequency f_s such that the continuous-time signal $x_a(t)$ can be perfectly recovered from its samples $x(n)$?

g) Is it possible to sample the spectrum $X(f)$ and to reconstruct $x(n)$ out of these spectral samples? Motivate your answer.

Assignment 3:

DC offset is an offsetting of a signal from zero. The term originated in electronics, where it refers to a direct current voltage, but the concept has been extended to any representation of a waveform. DC offset is the mean amplitude of the waveform; if the mean amplitude is zero, there is no DC offset.

DC offset is usually undesirable. For example, in audio processing, a sound that has DC offset will not be at its loudest possible volume when normalized, and this problem can possibly extend to the mix as a whole, since a sound with DC offset and a sound without DC offset will have DC offset when mixed. It may also cause other artifacts depending on what is being done with the signal. However, DC offset can be reduced in real-time by a one-pole one-zero high-pass filter.

Consider the following transfer function of a causal system which we will use to suppress the DC component:

$$H(z) = \frac{z-1}{z-a},$$

and its implementation shown below.

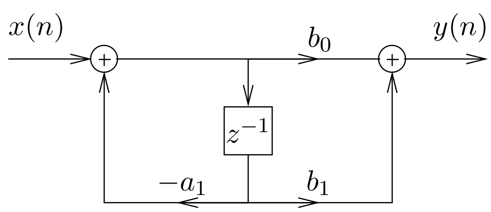

Figure 2: Block diagram of an implementation of $H(z)$.

a) What are the values of the filter coefficients b_0, b_1 and a_1?

b) Determine the poles and zeros of $H(z)$, and plot the pole-zero map.

c) Indicate how the stability of the system depends on the choice of a.

d) Sketch the corresponding magnitude and phase response when $a = 0.99$.

e) Compute the impulse response of the filter.

Consider the following (suddenly applied) input signal

$$x(n) = u(n),$$

and assume the system is initially in rest.

f) Compute the total response of the system $y(n) = y_{\text{tr}}(n) + y_{\text{ss}}(n)$, where $y_{\text{tr}}(n)$ and $y_{\text{ss}}(n)$ denote the *transient* and *steady-state* response of the system, respectively. Can you explain these results?

g) Alternatively, we can express the total response of the system $y(n) = y_{\text{zs}}(n) + y_{\text{zi}}(n)$, where $y_{\text{zs}}(n)$ and $y_{\text{zi}}(n)$ denote the *zero-state* and *zero-input* response of the system, respectively. Compute $y_{\text{zs}}(n)$ and $y_{\text{zi}}(n)$.