Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science
Information and Communication Theory Group

Stochastic Processes (ET3260IN/ET2505-D1)

Exam, Friday 24 October 2008, **9:00-12:00**.

Answer each question on a separate sheet.
Write your name and student number on <u>each</u> sheet.

Question 1 (10 points)

We consider the following joint-PDF of the random variables X and Y:

$$f_{X,Y}(x,y) = \begin{cases} ax & 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

(a. 1 pt) Determine the value of a.

(b. 2 pt) Calculate the marginal PDF of X and the marginal PDF of Y.

(c. 2 pt) Show that X and Y are uncorrelated random variables.

(d. 1 pt) Are X and Y independent random variables?

(e. 1 pt) Are X and Y orthogonal random variables?

(f. 1 pt) Calculate the expected value of and variance of random variable $Z = 3X - Y$

(g. 1 pt) Calculate the probability $P[Y \leq 1]$.

(h. 1 pt) Calculate the conditional joint probability density function $f_{X,Y|Y\leq 1}(x,y)$.

Please start answering the following question on a separate sheet!
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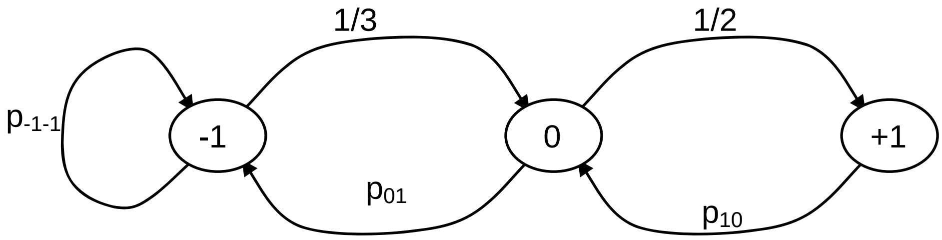
Question 2 (10 points)

A time-discrete amplitude-discrete random process X_n is modeled as a Markov chain with the following state transition diagram:



(a. 2pt) Which of the following series are <u>not</u> sample functions (realizations) that can be generated by X_n? Explain your answer

I.	0	-1	-1	0	0	1	0	1	0	1	0	-1
II.	-1	-1	-1	0	-1	0	-1	0	-1	0	-1	-1
III.	-1	-1	0	1	1	0	-1	-1	-1	0	-1	0
IV.	-1	-1	-1	-1	1	-1	-1	1	-1	-1	-1	-1
V.	0	1	0	1	0	1	0	1	0	1	0	1
VI.	-1	0	1	0	-1	0	1	0	-1	0	1	0

(b. 1pt) If $P[X_0=0]=1$, which of the two following realizations has a higher probability:

	n=0	n=1	n=2	n=3
I.	0	1	0	1
II.	0	–1	0	-1

(c. 1pt) Explain why X_n is WSS.

(d. 2pt) Show by calculation that the limiting state probabilities are given by

$P[X_n = -1] = 1/2$
$P[X_n = 0] = 2\, P[X_n = 1]$

(e. 2pt) Calculate $E[X_n]$.

(f. 2pt) Calculate $R_X(k)$ for $k = 0$, $k = 1$, and $k = -1$

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Question 3 (7 points)

We consider the linear processing of the wide sense stationary signal $X(n)$, with autocorrelation function,

$$R_X(k) = \begin{cases} 2 & k = 0 \\ 1 & |k| = 1 \\ 0 & otherwise \end{cases},$$

through a linear systems, specified by the filter $h(n)$ with impulse response

$$h(n) = \begin{cases} 2 & n = 0 \\ -1 & n = 1 \\ 0 & otherwise \end{cases}$$

The output of the linear system is given by $Y(n) = h(n) * X(n) = 2X(n) - X(n\text{-}1)$

(a. 1 pt) Show that the expected value of the random signal $X(n)$ is $E[X(n)] = 0$.

(b. 1 pt) Calculate the expected value of the random signal $Y(n)$, i.e. $E[Y(n)]$.

(c. 2 pt) Calculate the autocorrelation function of random signal $Y(n)$, i.e. $R_Y(n,k)$.

(d. 1 pt) Calculate the variance of the random signal $Y(n)$.

(e. 1 pt) Is $Y(n)$ a WSS random process? Motivate your answer.

(f. 1 pt) Is $Y(n)$ an IID random process? Motivate your answer.

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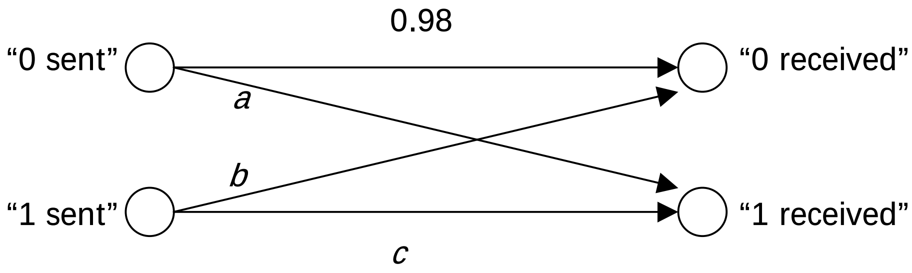
Question 4 (9 points)

We consider a binary communication channel with the following probabilistic channel model:



As can be seen from the model, the probability of receiving an "0" when an "0" is sent, equals 0.98. Further it is known that the probability of "0 sent" equals 0.4, and that the probability of "0 received" equals 0.410.

(a; 1p) Determine the conditional probability *a*: i.e.

$$P(\text{"1 } received\text{"} \mid \text{"0 } sent\text{"}).$$

(b; 2p) Determine the conditional probabilities *b* and *c*.

(c; 2p) Calculate the probability that a "0" was sent if a "0" has been received.

(d; 2p) Calculate the probability that an error is made when transmitting a single bit.

(e; 2p) Calculate the probability that more than forty bits are transmitted erroneously when we transmit 1000 bits over this channel.