Delft University of Technology Mekelweg 4
EWI Faculty 2628 CD Delft

Financial Mathematics WI4079
First Partial Exam
October 31st 2013, 9.30-12.30

Please provide your student card on the table, ready for inspection.
Mobiles, tablets and similar objects must be switched off.
During the first hour you cannot leave the room, even if you decide not to hand your exam in.
After the first hour, if you need to go to the toilet (max one person at a time) you have to temporarily hand your exam and mobile in.
The exam is invalidated if you cheat, use your mobile, etc.
Please write with a pen. Pencils are not accepted.
Please write your name, surname and student number. Do the same for any additional sheet of paper.

Complete solutions will be on Blackboard in the next days.

Good Luck!

Name: . **Surname:** . **Student Number:**

Questions

Please: Write clearly and try to be as complete as possible.
The different questions give different points. You can find the actual values in the brackets.
The total amount of points is 20.

1. Let $\mathcal{A}$ be the class of all the subsets of $\mathbb{R}$, which are finite unions of disjoint semi-open intervals like $(a,b]$ and $(c,+\infty)$. Enrich $\mathcal{A}$ by including the set $\emptyset$. Show that $\mathcal{A}$ is closed under finite intersections. (2 Points)

2. Let $\mathcal{A}$ be defined as in the text of the previous point. Consider all the intervals of type $(a,b]$ and $(c,+\infty)$ in $\mathbb{R}$. Let $F:\mathbb{R}\to[0,1]$ be a proper repartition function. Then define the following probabilities:
$$P_0((a,b]) = F(b) - F(a) \qquad \text{and} \qquad P_0((c,+\infty)) = 1 - F(c),$$
and
$$P_1(A) := \sum_{i=1}^{n} P_0((a_i,b_i]),$$
for $A \in \mathcal{A}$.
Show that P_1 is a probability on $\mathcal{A}$. (3 Points)

3. Let us assume that, for every $t_1,...,t_n \in T$, $n \geq 1$, we have defined $P^{t_1,...,t_n}$ on $(\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n))$. Let also assume that the following compatibility conditions hold:
   a. $P^{t_1,...,t_n}(A_1 \times ... \times A_n) = P^{t_{s_1},...,t_{s_n}}(A_{s_1} \times ... \times A_{s_n})$, where $(s_1,...,s_n)$ is any permutation of $(1,...,n)$, $n \geq 1$.
   b. $P^{t_1,...,t_n}(A \times \mathbb{R}) = P^{t_1,...,t_{n-1}}(A)$, $\forall A \in \mathcal{B}(\mathbb{R}^{n-1})$.

   Now let us define P on $(\mathbb{R}^T, \mathcal{B}(\mathbb{R}^T))$ such that
$$P(I_{t_1,...,t_n}(A)) = P\left((x_t)_{t\in T} \in \mathbb{R}^T : (x_{t_1},...,x_{t_n}) \in A\right) := P^{t_1,...,t_n}(A), \qquad A \in \mathcal{B}(\mathbb{R}^n),$$
   where $I_{t_1,...,t_n}(A)$ is a cylinder of basis A.
   Show that P is well-defined. (2 Points)
   (Hint: Assume that $I_{t_{i_1},...,t_{i_n}}(A) = I_{t_{j_1},...,t_{j_m}}(B)$, with $A \in \mathcal{B}(\mathbb{R}^n)$ and $B \in \mathcal{B}(\mathbb{R}^m)$.)

4. Let $\{B(t,\omega)\}_{t\geq 0}$ be a family of random variables, which are defined on a probability space $(\Omega, \mathcal{F}, P)$, with values in $\mathbb{R}$. When is the process $B(t,\omega)$ a Brownian motion? (2 Points)