Mid-term Exam Signal Processing TI2715-A/TI2710-A

October 4th, 2013

Question 1 (8 points in total)

An input-output system $\mathcal{S}_1$ is given by

$$y[n] = nx[n] + 2x[n-1].$$

(a) (2 p.) Proof whether or not the system is linear.

(b) (2 p.) Proof whether or not the system is time variant.

Another system $\mathcal{S}_2$ is given by

$$y[n] = x[n+1] + 2x[n-1].$$

(c) (1 p.) Give the impulse response $h[n]$ of this system and make a plot of $h[n]$.

(d) (1 p.) Explain why this is a non-causal system.

(e) (2 p.) Give the impulse response of a system that we can put in cascade with system $\mathcal{S}_2$ in order to make the total system causal.

Question 2 (8 points total)

A time-continuous sinusoidal signal is given by

$$x(t) = \cos(2\pi 100t + \pi/2) + 1.$$

(a) (1 p.) Choose a sampling frequency f_s. Motivate your answer.

(b) (2 p.) For the sampling frequency chosen under (a), give the expression for the resulting sampled time-discrete signal $x[n]$ and plot two periods of $x[n]$ as a function of n.

The samples of the time discrete-signal $x[n]$ are used to reconstruct a time-continuous signal $y(t)$.

(c) (1 p.) Choose an interpolation method. Then plot two periods of the resulting time-continuous interpolated signal y(t) as a function of t.

Now take the sampling frequency as $f_s = 100$ Hz. This will result in Aliasing.

(d) (2 p.) Make a plot of the time discrete-signal $x[n]$ when sampled with $f_s = 100$ Hz.

(e) (2 p.) What is the frequency of the reconstructed time-continuous signal $y(t)$ in this case?

Question 3 (6 points total)

Given is an LTI system in a black box $\mathcal{S}_1$ with unknown input-output relation.

(a) (1 p.) Explain how to determine the impulse response.

Assume we determine the impulse response of $\mathcal{S}_1$ to be:

$$h_1[n] = 4\delta[n] + \delta[n-1] - \delta[n-2].$$

(b) (1 p.) Give the input-output function in terms of input $x[n]$ and output $y[n]$ for this impulse response.

Given is an input $x_1[n] = \delta[n] - \delta[n-1]$.

(c) (2 p.) Determine output $y_1[n]$ by convolving $h_1[n]$ and $x_1[n]$.

Given is a second filter $\mathcal{S}_2$ with impulse response

$$h_2[n] = \delta[n] + \delta[n-1].$$

System $\mathcal{S}_1$ and $\mathcal{S}_2$ are put in cascade as indicated in Figure 1.

(d) (2 p.) Determine the overall impulse response $h[n]$ of the cascaded system.

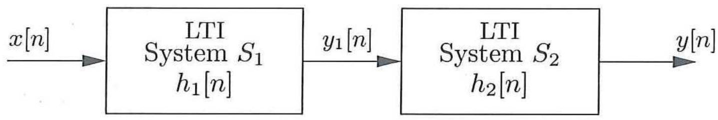

Figure 1: Cascaded system $\mathcal{S}_3$, composed of the LTI systems $\mathcal{S}_1$ and $\mathcal{S}_2$.