Exam Signal Processing TI2710-A/TI2715-A

November 5^{th}, 2013

Clearly indicate on your answer sheet whether you take TI2710-A (the old course) or TI2715-A (the new course including the laboratory exercises).

Question 2 - Frequency Transfer functions (9 points)

The frequency transfer function of an LTI system $\mathcal{S}$ is given by

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\left(2\cos(\hat{\omega}) + 1\right).$$

(a) (1 p.) Sketch the magnitude response of $H(e^{j\hat{\omega}})$ for $-\pi \leq \hat{\omega} \leq \pi$. Clearly mark the axes and indicate the zeroes and maxima in the sketch.

(b) (1 p.) Sketch the principal value of the phase response of $H(e^{j\hat{\omega}})$ for $-\pi \leq \hat{\omega} \leq \pi$.

(c) (1 p.) Show that $H(e^{j\hat{\omega}})$ can be written as

$$H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-2j\hat{\omega}}.$$

(d) (1 p.) The impulse response $h[n]$ that was used to obtain $H(e^{j\hat{\omega}})$ is given by

$$h[n] = \begin{cases} \alpha & \text{for } n = 0, \\ \beta & \text{for } n = 1, \\ \gamma & \text{for } n = 2, \\ \Delta & \text{elsewhere.} \end{cases}$$

Determine α, β, γ and Δ.

Consider the following input signal:

$$x[n] = 2 + \cos\left(\frac{2\pi}{3}n + \frac{\pi}{2}\right) + 2\cos(\pi n).$$

(e) (1 p.) Rewrite $x[n]$ in terms of complex exponentials and plot the complex spectrum. Clearly indicate the complex amplitudes $X(e^{j\hat{\omega}})$. *Hint: There is no need to compute any integrals to answer this question.*

(f) (2 p.) Determine the output $y[n]$ when $x[n]$ is used as input to system $\mathcal{S}$.

(g) (2 p.) Imagine that $H(e^{j\hat{\omega}})$ is applied to a signal that is sampled with a sampling frequency $f_s = 900$ Hz. Which frequency (in Hz) is then completely removed from the signal?

Question 4 - Fourier Series (6 points)

The Fourier series synthesis expression is given by

$$x(t) = \sum_{k=-N}^{N} a_k e^{j\frac{2\pi kt}{T_{fund}}}.$$

(a) (1 p.) How are the coefficients a_k and a_{-k} for $k \neq 0$ in general related?

Given is the following periodic signal $x(t)$ with fundamental period $T_{fund} = 2$,

$$x(t) = \cos(\pi t) + 2\cos(2\pi t).$$

The Fourier series analysis integral is given by

$$a_k = \frac{1}{T_{fund}} \int_0^{T_{fund}} x(t) e^{-j(2\pi kt/T_{fund})} dt.$$

(b) (1 p.) Based on $x(t)$, give the expression for a_2 using the analysis integral without computing the integral.

(c) (2 p.) For the given signal $x(t)$ we can also determine the complex amplitudes a_k using Euler's equation instead of computing the analysis integral. Show that $a_1 = 1/2$ and $a_2 = 1$, and, determine the value a_k for all other values of k.

(d) (1 p.) Plot the complex spectrum of $x(t)$. Clearly mark the scale of the axes of the two plots.

Another signal $z(t)$ is given by

$$z(t) = x(t) + 2.$$

(e) (1 p.) How do the coefficients a_k for signal $z(t)$ change compared to $x(t)$?