TU Delft

Delft University of Technology

Monte Carlo
Monday 23 Jan, 2017, 18.30-20.30

Three exercises. Eight questions. They all have equal payoff.

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1 Recall how Higham describes the Monte Carlo simulation of a financial derivative:

> The resulting Monte Carlo algorithm can be summarized as follows:
>
> for $i = 1$ to M
> compute an $\mathrm{N}(0, 1)$ sample ξ_i
> set $S_i = S_0 e^{(r-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}\xi_i}$
> set $V_i = e^{-rT}\Lambda(S_i)$
> end
> set $a_M = \left(\frac{1}{M}\right)\sum_{i=1}^{M} V_i$
> set $b_M^2 = \left(\frac{1}{(M-1)}\right)\sum_{i=1}^{M}(V_i - a_M)^2$
>
> The output provides an approximate option price a_M and an approximate 95% confidence interval (15.5).

A The 95 percent confidence interval is equal to

$$\left(a_M - 1.96\frac{b_M}{\sqrt{M}}, a_M + 1.96\frac{b_M}{\sqrt{M}}\right)$$

What is $\mathbf{P}(Z > 1.96)$ for a standard normal random variable Z? And why is this related to the confidence interval?

B The confidence interval is based on the fact that the random number a_M is approximately normally distributed. Why is this true? large numbers

2 A A Pareto random variable has distribution function $F(x) = 1 - \frac{1}{1+x}$ for $x \geq 0$. Write a small computer program that produces Pareto random numbers. You must use the standard random number generator `rand`.

B Consider the integral

$$\int_0^4 \frac{\sin(x)}{x\sqrt{x}}dx$$

Write a small computer program that computes this integral by a standard Monte Carlo simulation.

C It is possible to represent the integral in B as the expected value

$$\mathbf{E}\left[\frac{4\sin(X)}{X}\right]$$

What is the distribution function of X?

3 Consider the following pseudocode

for $i = 1$ to M
  for $j = 0$ to $N-1$
    compute an $\mathsf{N}(0,1)$ sample ξ_j
    set $S_{j+1} = S_j e^{(r-\frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\xi_j}$
    set $\overline{S}_{j+1} = \overline{S}_j e^{(r-\frac{1}{2}\sigma^2)\Delta t - \sigma\sqrt{\Delta t}\xi_j}$
  end
  set $S_i^{\max} = \max_{0\le j\le N} S_j$
  set $\overline{S}_i^{\max} = \max_{0\le j\le N} \overline{S}_j$
  if $S_i^{\max} < B$ set $V_i = e^{-rT}\max(E - S_N, 0)$, otherwise $V_i = 0$
  if $\overline{S}_i^{\max} < B$ set $\overline{V}_i = e^{-rT}\max(E - \overline{S}_N, 0)$, otherwise $\overline{V}_i = 0$
  set $\widehat{V}_i = \frac{1}{2}(V_i + \overline{V}_i)$
end
set $a_M = \frac{1}{M}\sum_{i=1}^{M}\widehat{V}_i$
set $b_M^2 = \frac{1}{M-1}\sum_{i=1}^{M}(\widehat{V}_i - a_M)^2$

A. Explain which financial derivative is simulated. Do you expect a bias in this simulation?

B. Which Monte Carlo method is used? Explain why it is helpful.

C. What would be a good control variate? Motivate your answer. You do not need to adapt the code.