Delft University of Technology

Monte Carlo
Monday 18 January, 2016, 18.30-21.30

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**Three exercises. They all have equal payoff.**

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**1** Higham's code `ch15.m` computes the Black-Scholes price of a European put:

```
%CH15     Program for Chapter 15
%
%   Monte Carlo for a European put
randn('state',100)
%%%%%%%%%% Problem and method parameters %%
S = 4; E = 5; sigma = 0.3; r = 0.04; T = 1;
Dt = 1e-3; N = T/Dt; M = 1e4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
V = zeros(M,1);
for i = 1:M
   Sfinal = S*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*randn);
   V(i) = exp(-r*T)*max(E-Sfinal,0);
end
aM = mean(V); bM = std(V);
conf = [aM - 1.96*bM/sqrt(M), aM + 1.96*bM/sqrt(M)]
```

A Adapt the code so that it uses antithetic variables.

B Adapt the code so that it computes the delta of a European put. There are several ways to do this, choose an optimal method.

C An up-and-out put only pays off if the asset price stays below a level $L$ until time of expiration. Suppose you want to compute the price of a up-and-out put for the same parameters as the code in `ch15.m` and at a level of $L = 6$. Explain how you have to adapt the code to compute an up-and-out put. Do you expect a bias?

D Can you use a control variate in exercise C? If so, which control variate? If not, why not?

**2** Let $X$ be a standard normal random variable. We want to compute the probability $\mathbf{P}(X < 20)$.

A Suppose we use standard Monte Carlo

$\mathtt{M} = 10^6;$

$\mathtt{Z} = \mathtt{randn}(\mathtt{M}, 1); \mathtt{X} = (\mathtt{Z} < 20);$

$\mathtt{aM} = \mathtt{mean}(\mathtt{X}); \mathtt{bM} = \mathtt{std}(\mathtt{X});$

$\mathtt{conf} = [\mathtt{aM} - 1.96 * \mathtt{bM}/\mathtt{sqrt}(\mathtt{M}), \mathtt{aM} + 1.96 * \mathtt{bM}/\mathtt{sqrt}(\mathtt{M})]$

What is the resulting confidence interval?

B The interval $[\texttt{aM} - \texttt{1.96} * \texttt{bM/sqrt(M)}, \texttt{aM} + \texttt{1.96} * \texttt{bM/sqrt(M)}]$ is the standard 95 percent confidence interval in our Monte Carlo computations (in the code above and in the code in exercise 1 as well). Explain what you need to do to make this a 90 percent confidence interval. Is this interval larger or smaller?

C The code in part A computes the integral $\int_{-\infty}^{\infty} h(x)f(x)dx$ where $f(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$ is the density function of the standard normal distribution. What is the function $h(x)$ in this case?

D According to the substitution rule $\int_{-\infty}^{\infty} h(x+20)f(x+20)dx$ is equal to $\int_{-\infty}^{\infty} h(x)f(x)dx$. Use this to adapt the code in part A, using $\texttt{Z} = \texttt{randn(M,1)} + \texttt{20}$ on line 2. Write down your code and mention what happens to the confidence interval.

**3** A. Suppose you use Monte Carlo to compute $\mathbf{E}[X]$ by using a control variate $Y$ with expected value $\mathbf{E}[Y] = 3.012$ and variance $Var(Y) = 1.872$. In your simulation it turns out that $\theta = 1.051$ is optimal, as used in $\mathbf{E}[X - \theta Y] + 3.012\theta$. Estimate the covariance between the random variables $X$ and $Y$.

B. Show that the samples of an exponential distribution with parameter $\lambda$ may be generated by $-\ln(\xi_i)/\lambda$ where the $\xi_i$ are $U(0,1)$ samples as in the random number generator `rand`.

C. Find a simple example where antithetic variates are less efficient than standard Monte Carlo.

D. Is it true that for any *non-negative* random variable $X$ the following inequality holds?

$$\mathbf{E}[X^3] \geq \mathbf{E}[X^2] \cdot \mathbf{E}[X]$$

If it is false, give a counterexample. If it is true, prove it!