TECHNISCHE UNIVERSITEIT DELFT
Faculteit Elektrotechniek, Wiskunde en Informatica

Exam IN4301 Advanced Algorithms

Part 2

November 22, 2017
16:00 - 17:00

- This is a closed book examination with 3 questions worth of 20 points in total.
- Your mark for this exam part will be the number of points divided by 2.
- This exam covers all information on the slides of the course of the first 4 lectures on approximation algorithms, and the set of papers as described in the study guide.
- Specify your name, student number and degree program, and indicate the total number of submitted pages on the first page.
- Write clearly, use correct English, and avoid verbose explanations. Giving irrelevant information may lead to a reduction in your score.
  Notice that almost all questions can be answered in a few lines!
- Use of book, readers, notes, and slides is not allowed.
- Use of (graphical) calculators is not permitted.
- If your average for all three exam parts is at least a 5.0, the average of the programming exercises, as well as the average of the homework exercises, your final mark for this course is the average of these three marks, rounded to the nearest half of a whole number. That is, 9.7 is rounded to 9.5, and 5.8 is rounded to 6.
- The total number of pages of this exam is 1 (excluding this front page).

1. Let A be an approximation algorithm for a problem P. Let $r_A(x)$ be the performance ratio of A for an instance $x \in P$, r_A the approximation ratio of A, and let ρ be the approximation threshold of P.
   (a) (1 point) Explain exactly whether or not it is possible that $r_A < \rho$.
   (b) (3 points) We happen to know that A is a c-approximation algorithm. What is the relation between (1) c and $r_A(x)$ for an instance $x \in P$, (2) c and r_A, (3) c and ρ?
   (c) (1 point) Suppose P is a maximisation problem and A is a 1.1-approximation algorithm for P. For a certain instance $x \in P$, one obtains $A(x) = 120$. What are the possible values for the optimal value $OPT(x)$ of the instance x?
   (d) (2 points) Let x_0 be a *tight example* of P for A. What do you know about the relation between $r_A(x_0)$, r_A, and ρ?

2. Consider the Load Balancing problem discussed in Lecture 5.
   (a) (2 points) Describe the LPT-algorithm we discussed in the lectures in at most three lines.
   (b) (3 points) Suppose we have a Load Balancing instance where it is known that
      1. the total processing time $\sum_{j=1}^{n} t_j$ of all the jobs involved equals 1400;
      2. the number m of machines is 2;
      3. the maximum processing time of a job is 7.

      Show that in this case the performance ratio of the LPT-algorithm is bounded above by 1.005 (Hint: what do you know about $\sum_{j=1}^{n} t_j/m$?)
   (c) (2 points) Someone states that a greedy approximation algorithm for PARTITIONING has been found with an approximation ratio $c = 1.5$. Point out to this person that after following the Advanced Algorithms Course, he could do better: Show that in fact there exists a greedy 7/6-approximation algorithm for this problem.

3. Consider MAX INDEPENDENT SET. Here, you are given an undirected graph G and you have to find a subset $V' \subseteq V$ of nodes as large as possible, such that no two nodes u, v in V' are directly connected via an edge in E. This problem is known to be NP-hard.
   (a) (2 points) Consider an instance $G = (V, E)$ of MAX INDEPENDENT SET. Let $G^2 = (V \times V, E')$, where $\{(u_1, u_2), (v_1, v_2)\} \in E'$ iff $\{u_1, v_1\} \in E$ or $\{u_2, v_2\} \in E$.
   Show that G has an independent set of size m iff G^2 has an independent set of size m^2.
   (b) (3 points) Using this construction of G^2 from G, show that if there exists a c-approximation algorithm for some finite $c > 1$ for MAX INDEPENDENT SET then there also exists a $c^{1/2}$-approximation algorithm for MAX INDEPENDENT SET.
   (c) (1 point) The construction of G^2 from $G = (V, E)$ in the previous sub questions can be easily extended to arbitrarily large $k \geq 2$ by defining
   $G^k = (V^k, E')$ where

   $$\{(u_1, u_2, \ldots, u_k), (v_1, v_2, \ldots v_k)\} \in E' \text{ iff } \{u_i, v_i) \in E \text{ for some } i = 1, 2, \ldots, k.$$

   By means of this construction it can be shown that whenever MAX INDEPENDENT SET has a c-approximation algorithm for some $c < \infty$, then MAX INDEPENDENT SET has a $c^{1/k}$-approximation algorithm.
   Using this information prove (exactly!) that MAX INDEPENDENT SET either belongs to PTAS or belongs to NPO - APX.

End of Exam Part 2