TU Delft
Calculus TI1106M – Final test
Monday 30-01-2017, 9:00 – 11:00

NAME: STUDENT ID:

Remarks:

- Formula sheet is allowed, calculators are not.
- In every problem, results from earlier problems can be used.

Short answer problems
Fill in anwers on sheet

4pt 1. Let $z = \dfrac{2\mathrm{i}}{4-\mathrm{i}}$.
Rewrite z in the form $a + b\mathrm{i}$ with a and b real.

2. Consider the following sequence: $a_n = \dfrac{3n-1}{\sqrt{2+n^2}}$ for $n \geq 1$ and integer.

2pt a. Indicate whether it is increasing, decreasing or neither.

2pt b. Find, if possible, $\lim_{n\to\infty} a_n$.
In case of divergence, write DIV.

4pt 3. Given vectors $\mathbf{v} = \langle 2, -3, -1\rangle$ and $\mathbf{w} = \langle 2, h, 2\rangle$.
Find h such that $\mathbf{v}$ and $\mathbf{w}$ are orthogonal.

4. Let function f be given by $f(x, y) = \sqrt{x^2 + xy}$, and point $P = (2, 6)$.

2pt a. Find $\nabla f(P)$.

2pt b. Find the directional derivative of f at P in the direction $\mathbf{v} = \langle 1, -2\rangle$.

2pt c. Use a linearization to approximate $f(2.04, 5.96)$

TURN PAGE

Open problems

Provide calculations and argumentation!

1. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3\, 3^{n+1}}(2x-5)^n$.

6pt a. Show that the radius of convergence is $\frac{3}{2}$ and the center of convergence is $\frac{5}{2}$.

5pt b. Find the interval of convergence.

4pt 2. a. Write the complex number $-\dfrac{1}{3} - \dfrac{\mathrm{i}}{\sqrt{3}}$ in polar form.

5pt b. Find all solutions to the equation $z^2 = -\dfrac{1}{3} - \dfrac{\mathrm{i}}{\sqrt{3}}$.
Provide your answers in the form $a + \mathrm{i}b$ with a and b in $\mathbb{R}$.

4pt 3. a. In class it was shown that $\cos(x) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ for all $x \in \mathbb{R}$.
Use this to show that (for $x \geq 0$):

$$\int x^2 \cos(\sqrt{x})\, dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+3}}{(n+3)(2n)!} = C + \frac{1}{3}x^3 - \frac{1}{4\cdot 2!}x^4 + \frac{1}{5\cdot 4!}x^5 - \frac{1}{6\cdot 6!}x^6 + \dots,$$

where C is an arbitrary constant.

6pt b. Use a. to approximate the integral $\displaystyle\int_0^1 x^2 \cos(\sqrt{x})\, dx$ with an error ≤ 0.01.
Motivate your answer!

4. Let f be a function of 2 variables given by $f(x,y) = y^3 - x^2 + \ln(y^3 x)$.
Let D be the maximal domain of f.

4pt a. Describe D and make a sketch. Clearly indicate or describe which points are part of D and which are not. Explain your answer.

4pt b. Show that $P = \left(-\sqrt{\frac{1}{2}}, -1\right)$ is a critical point of f.

6pt c. Explain whether f has a local maximum, local minimum or neither at P.

5pt d. Does f have any other critical points? Explain!

5. Consider the following iterated integral: $\displaystyle\int_{y=1}^{2} \int_{x=0}^{\sqrt{y}} f(x,y)\, dx\, dy$.
Let D be the domain of integration.

2pt a. Sketch D.

6pt b. To reverse the order of integration, the integral has to be split:

$$\iint_D f(x,y) dA = \int_{\dots}^{\dots} \int_{\dots}^{\dots} f(x,y)\, dy\, dx + \int_{\dots}^{\dots} \int_{\dots}^{\dots} f(x,y)\, dy\, dx.$$

Give the limits for both integrals.

6pt c. Evaluate $\displaystyle\iint_D y^{3/2} \cos(x\sqrt{y}) dA$.
Use the order of integration that you think is most suitable.

$$\text{Grade} = 1 + \frac{\text{score}}{9}$$