Midterm Reasoning and Logic (TI1306)

- The use of the book, notes, calculators or other sources is **strictly prohibited**.

Multiple choice questions

1. Which formula shows that both p and q are necessary for r?
   A. $((p \wedge q) \rightarrow r)$.
   B. $(\neg(p \wedge q) \vee \neg r)$.
   C. $(\neg r \vee (p \wedge q))$.
   D. $(r \rightarrow \neg(p \wedge q))$.

2. Suppose you create a truth table for A and B, both formulas in propositional calculus, and have a look at the columns below the main connectives of A and B. When do we know **for sure** that $A \equiv B$ is true?
   A. When there is a T in every row under B if there is a T in the same row under A.
   B. If there is no row for which there is an F under A and a T under B.
   C. If there is a T in every row under A and a T in every row under B.
   D. If there is no row in which there is only a T under A or B.

3. XOR (the exclusive OR) $(p \oplus q)$ is equivalent with the **negation** of:
   A. the bi-implication $(p \leftrightarrow q)$.
   B. the conjunction $(p \wedge q)$.
   C. the NAND $(p \mid q)$.
   D. the implication $(p \rightarrow q)$.

4. If p and q are propositions, what is the right way to prove that $(p \rightarrow q)$ is true?
   A. Assume p is true. (*Assume q is true.*) This means that $(p \rightarrow q)$ is true. QED
   B. Assume q is true. (*Now show that $\neg p$ is true*) This means that $(p \rightarrow q)$ is true. QED
   C. Assume q and $\neg p$ are true. (*Now deduce a contradiction*) This means that $p \rightarrow q$ is true. QED.
   D. Assume $\neg q$ is true. (*Deduce that $\neg p$ is true.*) This means $(p \rightarrow q)$ is true. QED.

5. Suppose $(\neg p \vee q)$ is true, $(q \rightarrow \neg r)$ is true, and r is true. What can we conclude?
   A. $(p \wedge \neg q)$ is true.
   B. $(p \rightarrow \neg q)$ is true.
   C. $(\neg p \wedge q)$ is true.
   D. $(\neg p \rightarrow q)$ is true.

6. Looking at the truth tables of formulas A, B, C in propositional calculus, when do you know for **sure** that the argument $A, B \therefore C$ is **not** logically valid?
   A. If there is a T under C in all rows where there is an F under A and B.
   B. If there is not a row for which there is a T under A, B and C.
   C. If there is an F under C in all rows where there is a T under A or B.
   D. If there is a row with a T under A and B, and an F under C.

7. Simon and Charles are students in Delft. When can they know for sure that the next statement is **false**? "If there exists a student in Delft without a bicycle, then all students in Delft have a car."
   A. If Simon does not have a bicycle, but Charles has a car.
   B. If Charles does not have a bicycle, and also no car.
   C. If Charles has a bicycle, and Simon a car.
   D. If Simon has a bicycle, but not a car.

8. If $P(x, y)$ is a property of a pair of two integers, how can we prove the next statement?
   "For all integers x there exists an integer y such that $P(x, y)$ is true.
   A. Find an integer x such that $P(x, y)$ is true for an arbitrary integer y.
   B. Take an arbitrary integer x. Find an integer y such that $P(x, y)$ is true.
   C. Find an integer x, and an integer y such that $P(x, y)$ is true.
   D. Take arbitrary x and y, and show that if $P(x, y)$ is true, then x and y cannot be integers.
9. How can we show that the statement in question 8 is false?
   A. Find an integer x and an integer y such that $P(x, y)$ is false.
   B. Take an arbitrary integer x, and find an integer y such that $P(x, y)$ is false.
   C. Find an integer x, such that $P(x, y)$ is false for an arbitrary integer y.
   D. Take arbitrary integers x and y, and show that $P(x, y)$ is false.
10. Which of the formulas below shows that there exists **exactly one** object with property P?
   A. $\exists x(P(x) \land \forall y(P(y) \to (x = y)))$
   B. $\exists x \forall y(P(x) \land (P(y) \to \neg(x = y)))$
   C. $\exists x(P(x) \land \exists y(P(y) \to \neg(x = y)))$
   D. $\exists x \exists y((P(x) \land P(y)) \to (x = y))$

Open questions

1. Suppose we have the following argument.

   Argument. $((p \land q) \to \neg r), (p \lor \neg q), (\neg q \to p) \therefore \neg r.$

   (a) (2 points) Create a truth table to investigate if the argument is logically valid or not. (You can use either $0/1$ or T/F. Order the rows in a systematic manner in any case.)
   (b) (1 point) Is the argument valid?
   (c) (1 point) Explain clearly why your answer at question b follows from your truth table at question a. Give numbers to all relevant columns in the truth table and refer to these numbers in your explanation.

2. Suppose we have the following arguments.

   Argument (I). *For all x and y: if both x and y are* irrational, $x \cdot y$ *is also* irrational.

   Argument (II). *For all integers n: if n^3 is* even, *then n is* even.

   (a) Argument I is not valid.
      i. (1 point) Give a counterexample which shows that argument I is not valid. (Tip: in the lecture a famous irrational number came by.)
      ii. (1 point) Explain clearly how your counterexample shows that the argument is not valid.
   (b) (2 points) Argument II is valid. Give a proof. Clearly justify the steps in your proof. Alternate these steps with an explanation of what should be proven. (Tip: contrapositive.)

End of the exam