Midterm Probability and Statistics TI2216M (4 October 2016, 18:30-20:30)

Only the use of a **non-graphical** calculator and a clean copy of the formula sheet is allowed.
This exam consists of 18 multiple choice questions.

Grade: Every correct multiple choice question counts for 0.5 points.

Explanation: Colour the boxes black or blue. Fill in the **version**, course code, your name and student number. The latter should be ticked as well. Finally, sign the sheet with your signature.

Version A

1. About the events A and B it is known that $P(A) = 0.2$, $P(B) = 0.5$ and $P(B|A) = 0.4$. Compute the probability $P(A \cup B)$.

   a. 0.73 **b.** 0.24 **c.** 0.46 **d.** 0.62 **e.** 0.55 **f.** 0.19

2. Assume the probability of having breast cancer for women complaining of hardening of breast is 0.01. Assume that the mammogram test is 90% accurate, that is, given that a woman has cancer, the test result is positive with probability 0.9 and given that a woman doesn't have cancer, the test result is negative with probability 0.9. What is the approximate probability that a woman (complaining of hardening breast) will have a positive test result?

   a. 0.05 **b.** 0.11 **c.** 0.25 **d.** 0.50 **e.** 0.75 **f.** 0.90

3. [continuation of the previous question] What is the approximate probability that a woman has cancer, given that the test result is positive?

   a. 0.04 **b.** 0.08 **c.** 0.27 **d.** 0.56 **e.** 0.74 **f.** 0.93

4. Toss two fair different standard dice, black and white. Consider three events:
   $A_1 = \{$ first die is $1, 2,$ or $3\}$, $A_2 = \{$ first die is $3, 4,$ or $5\}$,
   $A_3 = \{$ sum of faces of both dice is $9\}$. The probability $P(A_1 \cap A_2 \cap A_3)$ is equal to

   a. $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{1}{6}$ **d.** $\frac{1}{9}$ **e.** $\frac{1}{12}$ **f.** $\frac{1}{36}$

5. [continuation of the previous question] Which of the following statement(s) is (are) true?

   (i) $P(A_1 \cap A_2) = P(A_1)P(A_2)$, i.e. A_1 and A_2 are independent
   (ii) $P(A_1 \cap A_3) = P(A_1)P(A_3)$, i.e. A_1 and A_3 are independent
   (iii) $P(A_2 \cap A_3) = P(A_2)P(A_3)$, i.e. A_2 and A_3 are independent
   (iv) $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$.

   a. all four **b.** (i), (ii) and (iii) **c.** (i) and (iv)
   d. only (iii) **e.** only (iv) **f.** none of the four

6. This midterm consists of 18 multiple choice questions. All but two question have six possible answers, only one of which is correct. For simplicity assume that each question has six possible answers. G.G.M does not know the answer to any of the questions, so, for each question, (s)he selects the answer at random. What is the approximate probability that (s)he answers at least three questions correctly?

   a. 0.23 **b.** 0.54 **c.** 0.02 **d.** 0.60 **e.** 0.76 **f.** 0.87

7. Consider the following functions:

$$f(x) = \begin{cases} \frac{3}{x^3}, & x \geq 1, \\ 0, & \text{otherwise}, \end{cases} \quad g(x) = \begin{cases} \frac{2}{x^3}, & x \geq 1, \\ 0, & \text{otherwise}, \end{cases} \quad h(x) = \begin{cases} \frac{2}{x^3}, & \frac{1}{2} \leq x \leq 1, \\ 0, & \text{otherwise}. \end{cases}$$

Which of this (these) function(s) is/are a probability density?

a. Only h **b.** Only g **c.** f and h **d.** f and g **e.** g and h **f.** Only f

8. Compute the expectation of $Y = 2 - 3X^2$, if X is a uniform distribution on the interval $(0, 2)$.

a. 2 **b.** 0 **c.** -2 **d.** 6 **e.** -6 **f.** $\frac{7}{4}$

9. Assume X is a random variable and let $X_1 = 2X$, $X_2 = 2X + 1$, $X_3 = 2X + 3$, $X_4 = -2X + 1$ and $X_5 = -2X + 3$. Which of the following statement(s) is (are) true?

(i) The random variables $X_1, X_2, \ldots, X_5$ have the same expectation.
(ii) The random variables $X_1, X_2, \ldots, X_5$ have the same variance.
(iii) X_3 has the biggest variance, that is $\text{Var}(X_3) > \text{Var}(X_i)$, for all $i \in \{1, 2, 4, 5\}$.
(iv) X_3 has the biggest expectation, that is $\text{E}[X_3] > \text{E}[X_i]$, for all $i \in \{1, 2, 4, 5\}$.

a. all four **b.** (i) and (ii) **c.** (iii) and (iv) **d.** only (ii)
e. only (iv) **f.** none of the four

10. Let the random variable U have a $U(0, 1)$ distribution. We want to draw random numbers from a distribution with density

$$f(x) = \begin{cases} 2x - 2, & 1 \leq x \leq 2, \\ 0, & \text{otherwise}, \end{cases}$$

If Step 1: $U = rand;$, then Step 2:

a. $X = 1 + \sqrt{U}$ **b.** $X = 1 + U^2$ **c.** $1 + U$
d. $2 - U$ **e.** $2 - U^2$ **f.** $2 - \sqrt{U}$

11. Assume X and Y are two independent random variables, both exponentially distributed with parameter $\lambda = 1$. Let $M = \max\{X, Y\}$ be the maximum between the two random variables. Then the density of M is $f_M(x) = 0$, for $x < 0$, and for $x \geq 0$:

a. $f_M(x) = 1 - e^{-2x}$ **b.** $f_M(x) = (1 - e^{-x})^2$ **c.** $f_M(x) = 1 - (1 - e^{-x})^2$
d. $f_M(x) = 2e^{-2x}$ **e.** $f_M(x) = 2e^{-x} - e^{-x}$ **f.** $f_M(x) = 2e^{-x} - 2e^{-2x}$.

12. Let X and Y be two independent normally distributed random variables, where $X \sim N(1, 4)$ and $Y \sim N(5, 16)$. Let $W = 2X + 3Y + 4$. Then the variance of W is

a. 60 **b.** 56 **c.** 160 **d.** 100 **e.** 86 **f.** 0

13. Let X and Y be two discrete random variables with the joint probability mass function given in the table. Which of the following statement(s) is (are) true?

		X=a		
		0	1	2
	-1	1/6	1/6	1/6
Y=b	1	0	1/2	0

a. X and Y are uncorrelated and dependent
b. X and Y are uncorrelated and independent
c. X and Y are correlated and dependent
d. X and Y are correlated and independent

14. The joint probability density function of the pair (X, Y) is given by

$$f_{X,Y}(x, y) = \begin{cases} 2\mathrm{e}^{-(2x+y)}, & x \geq 0 \text{ and } y \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

The joint cumulative distribution function $F_{X,Y}(x, y)$ is, for $x \geq 0$ and $y \geq 0$

a. e^{-2xy} b. $\mathrm{e}^{(1-2x)(1-y)}$ c. $(1 - \mathrm{e}^{-2x})(1 - \mathrm{e}^{-y})$
d. $-\mathrm{e}^{-2x}(1 - \mathrm{e}^{-y})$ e. $(1 - \mathrm{e}^{-2x})(-\mathrm{e}^{-y})$ f. $-\mathrm{e}^{-x}(1 - \mathrm{e}^{-y})$

15. Assume that X is a positive continuous random variable. Which of the following statement(s) follow(s) from Jensen's inequality?

(i) $\mathrm{E}\left[\mathrm{e}^{X}\right] \geq \mathrm{e}^{\mathrm{E}[X]}$.
(ii) $\mathrm{E}\left[\log \sqrt{X}\right] \leq \log \sqrt{\mathrm{E}[X]}$.

a. both b. only (i) c. only (ii) d. none

16. Compute the 90% percentile of the normally distributed random variable X with expected value 1 and variance $\frac{1}{4}$.

a. 0.34 b. 3.48 c. 1.37 d. 4.56 e. 1.64 f. 3.48

17. Let X, Y and Z be any random variables (possibly with different distributions). Which of the following statement(s) is (are) always correct?

(i) $\mathrm{Cov}(aX, Y) = a\mathrm{Cov}(X, Y)$, for any $a \in \mathbb{R}$.
(ii) $\mathrm{Cov}(a_1X + b_1, a_2Y + b_2) = a_1a_2\mathrm{Cov}(X, Y)$ for any $a_1, a_2, b_1, b_2 \in \mathbb{R}$.
(iii) $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$.

a. all three b. (i) and (ii) c. (i) and (iii) d. (ii) and (iii)
e. only (iii) f. none of the four

18. Assume the customers arriving at EWI's canteen can be modelled by a Poisson process with intensity $\lambda = 120$ per hour. What is the probability that 10 customers arrive at the canteen between 12:20 and 12:30?

a. $\frac{\mathrm{e}^{-10} \cdot 10^{20}}{20!}$ b. $\frac{\mathrm{e}^{-20} \cdot 10^{20}}{10!}$ c. $\frac{\mathrm{e}^{-20} \cdot 10^{20}}{20!}$ d. $\frac{\mathrm{e}^{-10} \cdot 20^{10}}{10!}$ e. $\frac{\mathrm{e}^{-20} \cdot 10^{20}}{10!}$ f. $\frac{\mathrm{e}^{-20} \cdot 20^{10}}{10!}$