TECHNISCHE UNIVERSITEIT DELFT
Faculteit Elektrotechniek, Wiskunde en Informatica

TUDelft

Algebra 1 (TW1061)

11 March 2019, 9.00 – 11.00 (2 hours).

Responsible examinator: dr. D.C. Gijswijt.

This exam consists of **5 problems** worth 10 pts in total.
The grade is equal to the number of obtained pts.
No books, written notes, calculators, or mobile phones are allowed during the exam.
Please write your name on every sheet. **Good luck!**

1. The prime factorisation of 2019 is given by $2019 = 3 \cdot 673$. Let $G = (\mathbb{Z}/2019\mathbb{Z})^*$.

[1pts] (a) How many elements does G have?

[1pts] (b) Determine the inverse of $\overline{100}$ in G.

$a^{-1} = a$

[2pts] 2. Let G be a group with identity element e. Suppose that $a^2 = e$ for every $a \in G$. Prove that G is abelian.

[2pts] 3. Give a homomorphism $f : V_4 \to \mathbb{R}^*$ that is not the trivial homomorphism.

[2pts] 4. Consider the homomorphism $f : \mathbb{C}^* \to \mathbb{C}^*$ given by $f(z) = \frac{z}{|z|}$.
Show that f is indeed a homomorphism and determine the kernel and image of f.

[2pts] 5. Consider the set

$$G = \left\{ \begin{bmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} : a, b, c \in \mathbb{R} \right\},$$

with the usual matrix multiplication as operation. Show that G is a group. Is G abelian?