Technische Universiteit Delft
Faculteit Electrotechniek, Wiskunde en Informatica

Mekelweg 4
2628 CD Delft

Monte Carlo methods (wi3425tu)
January 22nd 2019, 18.30–21.30 uur

(No books, no notes.)

Instructions:

- Answers should be supplemented by a motivation, explanation and/or calculation, whichever is appropriate.
- When reporting estimates based on a simulation, this is always understood to be: estimate plus standard error.
- You are expected to adhere to the conventions, known as *Points of attention for Monte Carlo simulations.*
- Questions involving coding: set the seed; the code for each part of a question should be saved in a separate file; when it is run, it must produce the results you reported; use the following convention for the filenames: a "Q," the exact question number, and then the extension ".m"; for example, the Matlab file for question 1c is `Q1c.m`.
- Point distribution: each (part of a) question carries the same weight; there are 12 parts.

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1. A random variable X has distribution function F with $F(x) = 0$ for $x < 0$, and $F(x) = 1 - 1/\sqrt{1+x}$ for $x \geq 0$. Describe how to construct X from a $U(0,1)$ random variable.

2. A simulation with 10^3 replications is performed, resulting in a standard error of 0.017. How many replications would you do, if you wanted a standard error smaller than 0.001?

3. **a.** A random variable X can be generated via $X = \sqrt[3]{-\ln U}$. Write a program to estimate $I_1 = \mathbb{E}[\sin(3X)]$, using antithetic variates; use $M = 10^4$. How much more efficient is the antithetic version compared to ordinary Monte Carlo, taking into account that the work (approximately) doubles?

b. Consider $I_2 = \mathbb{E}\left[e^{-X}\right]$. Compared to ordinary Monte Carlo, will antithetic variates result in a more efficient simulation? Explain and answer without doing any simulation.

4. Higham's code `ch19.m` is given. Adapt the code to value a fixed strike lookback put option, payoff $\max(E - S_{\min}, 0)$, where $S_{\min} = \min_{0 \leq t \leq T} S(t)$. Parameters: $S_0 = 60$, $E = 50$, $\sigma = 0.25$, $r = 0.05$, $T = 1$, $M = 10^4$.

a. Compute an estimate and standard error for the option price. Motivate your choice for the gridsize.

b. Is there bias? If yes: positive or negative; or impossible to tell? Explain.

c. Modify the program to use an ordinary put with the same strike as a control variate, using $\theta = 2$. Report the results. The Higham script-file `ch08.m` may be useful.

d. Determine the optimal θ-value and then redo the simulation. If we measure in terms of the number of replications needed, how much is the increase in efficiency?

e. Adapt the code from **a** and produce an estimate for the delta of this option; also report your choice for h.

5. Consider a down-and-in call option with partial barrier; the contract is only knocked-in if the stock price goes below the barrier on the interval $[0, T/2]$. For the questions below, start from the code in `Q5.m`, which is a stripped version of `ch19.m`, setup to apply importance sampling. Option parameters: $S_0 = 40$; $r = 0.05$; $\sigma = 0.3$; $E = 50$; $B = 35$; $T = 1$. Use $M = 10^4$ and $\Delta t = 10^{-3}$.

a. Modify the program to make it do ordinary Monte Carlo (so `mu` remains set to 0) for this exotic option; change the seed. Do an ordinary Monte Carlo simulation to price the option and report the results.

b. Recall the equation from the asset-price model: the asset value at time $T_1 = N_1 \Delta t$ (not necessarily "expiration") is given by

$$S(T_1) = S_0 \cdot \exp\left((r - \tfrac{1}{2}\sigma^2) \cdot T_1 + \sigma \sum_{j=1}^{N_1} \sqrt{\Delta t} \cdot Z_j \right).$$

If, for importance sampling, we apply a shift μ to Z_j, this can be accomplished by adding μ to each Z_j. From this formula an equation can be obtained to compute a drift value μ so that the median of $S(0.3T)$ is on the barrier. Determine and report this μ and implement it in the program. On $[0.3T, T]$ set $\mu = 0.055$; this will put the median at expiry near the strike (given a correct drift on the first part, of course). Run the simulation and report the results.

c. Give your assessment: are your importance sampling results to be trusted? Explain—you might include a sketch of a histogram for illustration.