Midterm Mathematical Structures TW1010
Monday November 5, 2018, 9:00-11:00

TU Delft

No calculators allowed. Write the solutions in the fields provided. The grade is (score+6)/6.

Exercise [ ] continued (extra space)

1 For each of the following, prove or give a counterexample

1a $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R} : x > y$ 2

1b $\exists y \in \mathbb{R}\ \forall x \in \mathbb{R} : x > y$ 2

2 Give the logical tautology which underlies the structure of the following proof. Also be specific which statements the variables p, q, etc. represent. 3

Theorem: If n is a prime number, then either $n = 2$ or n is odd.

Proof: Suppose n is a prime number which is not odd. Then n is even, so n is a divisible by 2. As any prime number is divisible only by 1 and itself, n must equal 2. Thus we have shown that any prime number is either 2 or odd.

3 Give an example of a relation on $\mathbb{N}$ which is reflexive, symmetric, but not transitive. 7
Be sure to show the example you describe satisfies these properties.

- My example relation is defined as

- The relation is **reflexive** as

- The relation is **symmetric** as

- The relation is **not transitive** as

4 Suppose $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(f(x)) = 5x + 4$. Show that f is injective. 4

5 Given a function $f : A \to B$ and a subset $C \subseteq A$.

5a Complete the definition: The image $f(C)$ equals 2

5b Show that $f(f^{-1}(f(C))) = f(C)$. 5

6 Define the sequence (F_n) recursively by $F_1 = 1$ and $F_2 = 1$ and $F_{n+2} = 2F_{n+1} + 3F_n$ for $n \geq 1$. Show that for all $n \in \mathbb{N}$ we have 8

$$F_1 + F_2 + F_3 + \cdots + F_n = \frac{1}{4}(F_{n+2} - F_{n+1})$$

7 Show that $\lim_{n\to\infty} \frac{4n^2+n+3}{2n^2-n} = 2$ using the definition of limit of a sequence. 8

8 Suppose (s_n) is a convergent sequence with $s_n \geq 0$ for all n, and $\lim_{n\to\infty} s_n = s > 0$. Show that $\lim_{n\to\infty} (s_n)^{\frac{1}{3}} = s^{\frac{1}{3}}$. 6

Hint: The equation $(a-b)(a^2+ab+b^2) = a^3 - b^3$ might be useful.

Remark: You are not allowed to use continuity in your solution.

9 Show that if (s_n) satisfies $\lim_{n\to\infty} s_n = \infty$, then for any $k \in \mathbb{R}$ we have $\lim_{n\to\infty}(s_n + k) = \infty$ as well. 5

10 Complete the definition: The sequence (s_n) is bounded if 2

Examiner responsible: Fokko van de Bult
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