Test 2 Mathematical Structures TW1010
Friday December 14, 2018, 10:45-11:45

No calculators allowed. Write the solutions in the fields provided. The grade is (score+4)/4.

Exercise [ ] continued (extra space)

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1 The sequence (s_n) is defined recursively as $s_1 = 0$ and $s_{n+1} = \frac{1}{3-s_n}$ for $n \geq 1$.

1a Show that $0 \leq s_n \leq 2$ for all $n \in \mathbb{N}$. 4

1b Show that the sequence (s_n) converges. 5

1c Obtain the limit $\lim s_n$. 3

The axioms of an ordered field as applied to $\mathbb{R}$ are

- A1 $\forall x, y \in \mathbb{R} : x + y \in \mathbb{R}$ and $x = w \wedge y = z \Rightarrow x + y = w + z$;
- A2 $\forall x, y \in \mathbb{R} : x + y = y + x$;
- A3 $\forall x, y, z \in \mathbb{R} : x + (y + z) = (x + y) + z$;
- A4 $\exists 0 : \forall x \in \mathbb{R} : x + 0 = x$ and this 0 is unique;
- A5 $\forall x \in \mathbb{R} : \exists(-x) \in \mathbb{R} : x + (-x) = 0$ and $(-x)$ is unique;
- M1 $\forall x, y \in \mathbb{R} : x \cdot y \in \mathbb{R}$ and $x = w \wedge y = z \Rightarrow x \cdot y = w \cdot z$;
- M2 $\forall x, y \in \mathbb{R} : x \cdot y = y \cdot x$;
- M3 $\forall x, y, z \in \mathbb{R} : x \cdot (y \cdot z) = (x \cdot y) \cdot z$;
- M4 $\exists 1 \neq 0 : \forall x \in \mathbb{R} : x \cdot 1 = x$ and this 1 is unique;
- M5 $\forall x \neq 0 : \exists(1/x) \in \mathbb{R} : x \cdot (1/x) = 1$ and $(1/x)$ is unique;
- DL $\forall x, y, z \in \mathbb{R} : x \cdot (y + z) = x \cdot y + x \cdot z$;
- O1 For all $x, y \in \mathbb{R}$ exactly one of $x = y$, $x > y$, holds $x < y$;
- O2 $\forall x, y, z \in \mathbb{R} : x < y \wedge y < z \Rightarrow x < z$;
- O3 $\forall x, y, z \in \mathbb{R} : x < y \Rightarrow x + z < y + z$;
- O4 $\forall x, y, z \in \mathbb{R} : x < y \wedge 0 < z \Rightarrow xz < yz$.

2 Assume $0 < x, y, z, w$. Show using the axioms that if $x < y$ and $w < z$ then $xw < yz$. Be sure to state which axiom you use in each step! 6

3 Finish the definition. A sequence (s_n) is called a Cauchy sequence if 2

4 Consider the sequence (s_n) defined by $s_n = \cos(\frac{n\pi}{6})$. Determine $\limsup s_n$. Be sure to precisely show that your answer is correct. 8

5 Given a non-empty, bounded set $S \subseteq (0, \infty)$. Let $T = \{1/s \mid s \in S\}$ be the set of all reciprocals[1] of elements from S.

5a Show that T has an infimum 2

5b Show that $\inf(T) = 1/\sup(S)$. 6

Examiner resposible: Fokko van de Bult
Examination reviewer: Wolter Groenevelt, Rik Versendaal.

[1] Reciproval betekent inverse van een getal in het Nederlands