Technische Universiteit Delft
Faculteit Elektrotechniek, Wiskunde en Informatica
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Midterm exam Kaleidoscope, (TW1021 and TW1021A)
5 October 2018, 9:00 - 11:00.

The use of a mobile phone or a calculator is forbidden.

Every answer has to be motivated.

(3) **1.** (a) How many solutions (x, y, z, w) does the equation $x + y + z + w = 20$ have, for which x, y, z, w are nonnegative whole numbers?

(2) (b) And how many for which x, y, z, w are positive whole numbers?

(3) (c) Prove that the number of solutions (x, y, z, w) of the equation $3x+y+z+w = 20$ for which x, y, z, w are nonnegative whole numbers is equal to

$$\sum_{k=0}^{6} \binom{22-3k}{2}.$$

(3) **2.** Prove that

$$\binom{n}{2} + \binom{n+1}{2} = n^2 \text{ for } n \in \mathbb{N},\ n \geq 2.$$

3. You want to send postcards to sixteen friends. The giftshop has four different postcards for sale.

(2) (a) Assuming there are enough postcards in the shop: in how many ways can you send postcards in such a way that each friend gets (exactly) one postcard.

(3) (b) The shop has exactly sixteen postcards left, four of each kind; in how many ways can you send one postcard to each of your friends.

(3) (c) Assuming there are enough postcards in the shop: in how many ways can you send postcards in such a way that each friend gets one or more different postcards.

Mark: (total number of points + 4)/4 rounded up to the nearest half.