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Exam Maple and Matlab, Kaleidoscope TW1021
2 November 2018, 13:30 - 15:30.

Preparation. First of all, read the instructions on the separate "Instruction page". Do not start with the exam before you have copied the .mw and .m files to the P: drive and have changed their names as explained therein.

Maple

- Fill in your name and student number at the top of the .mw file.
- Do not change the Maple commands already placed in the sections **Exercise 1.**, **Exercise 2.** and **Exercise 3.**.
- Use only the packages mentioned in the first lines of the sections, don't import extra packages.
- Answer the questions of exercise 1 in section **Exercise 1.**, the questions of exercise 2 in section **Exercise 2.** and the questions of exercise 3 in section **Exercise 3.**. Create extra paragraphs if needed.
- Are you ready? Delete superfluous Maple commands and text lines.

Matlab

- Fill in your name and student number at the top of the .m file.
- Do not change the Matlab commands already placed in the sections **Exercise 1.** and **Exercise 2.**.
- Answer the questions of exercise 1 in section **Exercise 1.** and the questions of exercise 2 in section **Exercise 2.**.
- Are you ready? Delete superfluous Matlab commands and text lines.
- Be sure that the sections are runnable and that Matlab doesn't give any warnings.

Your personal drive P: has to contain the files: Exam... .mw, Exam... .m, funcf.m, funcg.m funcdiff.m, trap_for.m and trap.m. Check this!

The use of a mobile phone or a calculator is forbidden.

The following questions has to be answered using Maple.

1. The function $f : (0, \infty) \to \mathbb{R}$ is given by $f(x) = \ln(\sqrt{x})$. This function has an inverse function $f^{inv} : \mathbb{R} \to (0, \infty)$. The graph of g is obtained from the graph f^{inv} by shrinking it vertically by a factor of 2.

(a) Use Maple commands to find an expression for f^{inv}.

(b) Plot the graphs of f and g, in the colors black and red, in one picture. Show them in the square $[-1, 5] \times [-1, 5]$.

Every vertical line to the right of the y-axis intersects the graph of f in a point P and the graph of g in a point Q. The length of the line segment PQ depends on the position of the vertical line.

(c) Calculate the minimal length of PQ.

2. The functions f and g are given by: $f(x) = -6x^2 + 12x$ and $g(x) = \frac{x^2+1}{x}$.

(a) Plot the graphs of f and g, in the colors black and red, in one picture. Show them in the square $[-10, 10] \times [-10, 10]$.

The graphs of f and g enclose a bounded region G.

(b) Approximate the area of G.

(c) Plot the graph of g and the line l with equation $y = x$ in one picture. Use different colors. Prove that l is a slant asymptote of the graph of g.

3. Consider the differential equation:

$$\frac{\mathrm{d}u}{\mathrm{d}t} + \frac{u}{2} = \mathrm{e}^{\frac{t}{3}} \tag{1}$$

(a) Use the `Maple Help` to get information about the command `DEplot`.
Plot a direction field for the differential equation (1) in the square $[0, 6] \times [-2, 4]$. The arrows has to be blue and of medium size, the scaling has to be constrained.

(b) Plot the direction field of (1) together with the solutions of the initial value problems:

$$\begin{cases} \frac{\mathrm{d}u}{\mathrm{d}t} + \frac{u}{2} = \mathrm{e}^{\frac{t}{3}} \\ u(0) = k \end{cases} \qquad k = -2, -1, 0, 1, 2. \tag{2}$$

The arrows of the direction field should be blue and of medium size, the graphs of the solutions should be black.
Hint: Create a sequence of initial values and again use the Maple command `DEplot`.

(c) Solve the initial value problem exactly:

$$\begin{cases} \frac{\mathrm{d}u}{\mathrm{d}t} + \frac{u}{2} = \mathrm{e}^{\frac{t}{3}} \\ u(3) = 0 \end{cases} \tag{3}$$

using the Maple command `dsolve`. The solution is an expression in t, convert it to a function v. Calculate $v(4)$ and round your answer to 4 decimal places.

The use of a mobile phone or a calculator is forbidden.

The following questions has to be answered using Matlab.

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1. The functions f and g are given by $f(x) = -6x^2 + 12x$ and $g(x) = \frac{x^2+1}{x}$.

(a) Write two function files `funcf.m` and `funcg.m`, with declaration statements `y=funcf(x)` and `y=funcg(x)`. The input variable `x` is an array and the output variable `y` the array of function values of `x`.

(b) Plot the graphs of the functions f and g on the interval $[-10, 10]$, with labels along the axes, a title and a legend as shown in FIGURE 1.

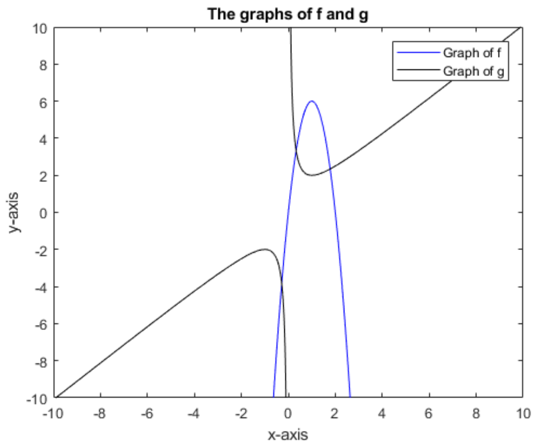

FIGURE 1

(c) Write a function `funcdiff.m`, with declaration statement `y=funcdiff(x)`, which calculates the difference of f and g. Use the function files made before (See: (a)).

(d) Use the Matlab function `fzero` to approximate the coordinates of the three intersection points. Use `Matlab Help` to get information about the usage of this function.

(e) Create a new plot of the functions f and g with the intersection points marked as red $*$ and an extended legend as shown in FIGURE 2. If we want to calculate area of the bounded region G,

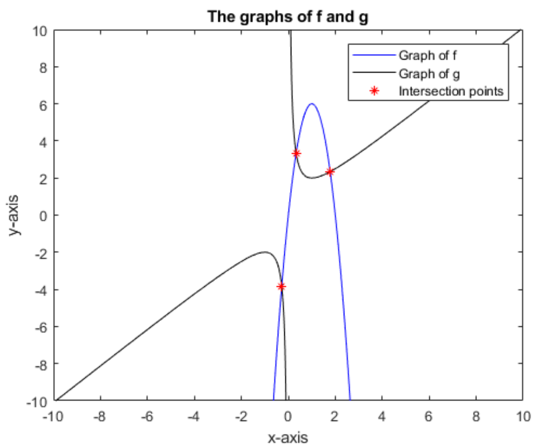

FIGURE 2

enclosed by the graphs of f and g, the `Trapezoid Rule` can be used.

Intermezzo about the Trapezoid Rule

Let f be a function on $[a, b]$ and let N be a positive integer, $h = \frac{b-a}{N}$, $x_n = a + nh \qquad (n = 0, 1, \ldots, N)$. Then the `Trapezoid Rule` is given by

$$\begin{aligned} &\sum_{n=1}^{N} \frac{f(x_{n-1}) + f(x_n)}{2} h \\ &= h \sum_{n=1}^{N} \frac{f(x_{n-1}) + f(x_n)}{2} \\ &= h \left(\sum_{n=0}^{N} f(x_n) - \frac{f(a) + f(b)}{2} \right) \end{aligned} \tag{1}$$

which is an approximation of $\int_a^b f(x)\,\mathrm{d}x$.

(f) Write two function files `trap_for.m` and `trap.m`, with declaration statements `tr=trap_for(fun,a,b,N)` and `tr=trap(fun,a,b,N)`. Both function files have to apply the Trapezoid Rule on the function `fun` where the interval $[a, b]$ has to be divided in N subintervals. Implement (1) and use a for-loop in the first function file and not a for-loop in the second one.

(g) Use the functions `trap_for.m` and `trap.m` to approximate the area of G (Choose `fun = @funcdiff` and `N = 1000`). Show the approximations in the 'Command Window'.

2. To measure the takeoff performance of an airplane, the horizontal position of the airplane was measured every second, from t = 0 to t = 12. The positions (in feet) were: 0, 8.8, 29.9, 62.0, 104.7, 159.1, 222.0, 294.5, 380.4, 471.1, 571.7, 686.8 and 809.2. The relation between the horizontal position of the airplane (p) and time (t) is given by: $p = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$.

(a) Create, with the given data, a matrix A and a vector $\mathbf{b}$ such that $\mathbf{x} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{b}$.

(b) Use the `Least squares method` (Matlab command: `\`) to find β_0, β_1, β_2 and β_3.

(c) Plot the graph of p as function of t on the interval $[0, 12]$ together with the given data such as in Figure 3.

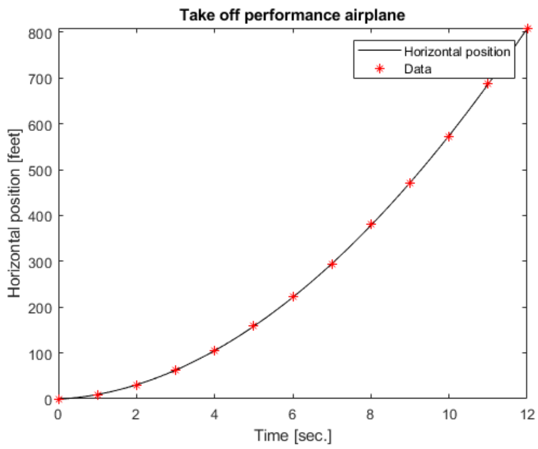

Figure 3