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\documentclass[a4paper,10pt,reqno]{amsart}
%
% packages
%
\usepackage{amssymb}
%
% definitions
%
\newtheorem{thm}{Theorem}[section]
\theoremstyle{definition}
\newtheorem{ex}{Exercise}[section]
%
% new commands
%
\newcommand{\NN}{\mathbb{N}}
\newcommand{\ee}{\mathrm{e}}

\begin {document}
\author{Ingeborg Goddijn}
\title{Exam LaTeX{}}
\date{\today}
\maketitle

\section{Mathematical Structures}
\begin{thm}
$p\Rightarrow q$ and $\sim q \Rightarrow \sim p$ are logically equivalent.
\end{thm}

\begin{proof}
The following truth table shows that $p\Rightarrow q$ and $\sim q \Rightarrow \sim p$
are logically equivalent.

\begin {center}
\begin {tabular}{c|c|ccccc}
\hline
$p$  &  $q$  & $(p \Rightarrow q)$ &$\Leftrightarrow$ &$[(\sim q)$ &$\Rightarrow$
&$(\sim p)]$ \\
\hline
T & T & T &T& F &F &F \\
T & F & F &T& T &F &F \\
F & T & F &T& F &T &T \\
F & F & T &T& T &T &T \\
\hline
\end {tabular}
\end {center}
\end{proof}

\begin{ex}
Prove by induction that
\begin{equation}\label{eq:1}
1+2+3+\cdots+n = \frac{1}{2}n(n+1) \mathrm{\ for\ all\ } n\in\NN.
\end{equation}
\end{ex}

\begin{proof}
Let $P(n)$ be statement~\eqref{eq:1}.
Then $P(1)$ asserts that $1 =\frac{1}{2}\cdot 1\cdot(1+1)$, $P(2)$ asserts that
$1+2=\frac{1}{2}\cdot 2\cdot(2+1)$, and so on.
In particular we see that $P(1)$ is true, and this establishes the basis for the
induction.

To verify the induction step, we suppose that $P(k)$ is true, where $k\in\NN$. That is,
we assume
$$1+2+\cdots+k=\frac{1}{2}k(k+1).$$
Since we wish to conclude that $P(k+1)$ is true, we add $k+1$ to both sides to obtain
\begin{align*}
1+2+3+\cdots+k+(k+1)&=\frac{1}{2}k(k+1)+(k+1) \\
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                    &=\frac{1}{2}[k(k+1)+2(k+1)]\\
                    &=\frac{1}{2}(k+1)(k+2)\\
                    &=\frac{1}{2}(k+1)[(k+1)+1].
\end{align*}
This last equation is $P(k+1)$, so $P(k+1)$ is true whenever $P(k)$ is true, and by the principle of mathematical induction we conclude that $P(n)$ is true for all $n\in \NN$.
\end{proof}

\section{Analysis I}
\begin{ex}
Prove that
\begin{equation*}\label{eq.Euler}
\ee^x =\sum_{n=0}^\infty\frac{x^n}{n!}=
\lim_{n\to\infty}\left(\frac{1}{0!}+\frac{x}{1!}+\frac{x^2}{2!}+
\cdots+\frac{x^n}{n!}\right).
\end{equation*}
\end{ex}

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