Technische Universiteit Delft
Faculteit Elektrotechniek, Wiskunde en Informatica
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Midterm exam Kaleidoscope, (TW1021 and TW1021A)
5 October 2018, 9:00 - 11:00.

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(3) **1.** (a) Every letter corresponds to a box and the right hand side of the equation corresponds to 20 indistinguishable balls to be distributed over the boxes. Every distribution creates a sequence of $20+3=23$ symbols (balls and separations between boxes) and every such sequence creates a distribution. As soon as we have placed 3 separations (between the boxes) we have found a possible distribution of 20 indistinguishable balls over 4 boxes. There are $\binom{23}{3}=1771$ of such distributions.

(2) (b) The question is nearly the same. First put 1 ball in each box and distribute the 16 other balls arbitrary. So the answer is $\binom{19}{3}=969$.

(3) (c) The equation can be written as $y+z+w=20-3x$ for $x \in \{0, 1, \ldots, 6\}$. As before the number of solutions of this equation can be found, $\binom{20-3k+2}{2}=\binom{22-3k}{2}$ for $k \in \{0, 1, \ldots, 6\}$. Summation over k gives the answer.

(3) **2.** $\binom{n}{2}=\frac{n!}{2!(n-2)!}$ and $\binom{n+1}{2}=\frac{(n+1)!}{2!(n-1)!}=\frac{(n+1)n!}{2!(n-1)(n-2)!}$ so
$\binom{n}{2}+\binom{n+1}{2}=\frac{n!}{2!(n-2)!}(1+\frac{n+1}{n-1})=\frac{n!2n}{2!(n-2)!(n-1)}=\frac{2n!n}{2(n-1)!}=n^2$ for $n \in \mathbb{N}$, $n \geq 2$.

(2) **3.** (a) There are $\binom{4}{1}=4$ ways to choose one postcard out of four. Because I want to send a postcard to sixteen friends there are 4^{16} ways to do so (for each of them I choose one out of four).

(3) (b) I number the four different postcards from 1 to 4. There are $\binom{16}{4}$ ways to send postcard 1 to four friends, $\binom{12}{4}$ ways to send postcard 2 to four friends, $\binom{8}{4}$ ways to send postcard 3 to four friends and $\binom{4}{4}=1$ way to send postcard 4 to four frends. So there are are $\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4}=\frac{16!}{(4!)^4}$ ways to send the sixteen postcards to sixteen friends.

(3) (c) Because I have four different cards , there are 2^4 ways to send different postcards to a friend (every card may or may not be sent). But then it is possible to send no postcard at all to this friend so there are 2^4-1 ways to send at least one postcard to him or her and all the postcards are different. If I want to do this for every of my sixteen friends I can do this in $(2^4-1)^{16}$ ways.

Mark: (total number of points + 4)/4 rounded up to the nearest half.