▼ Exercise 1.

restart;
with(*plots*);

$[$ *animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d,* **(1.1)**
conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot,
display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot,
implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot,
listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple,
odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot,
polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors,
setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot,
textplot3d, tubeplot $]$

$eq := y = \ln\left(\sqrt{x}\right)$;

$$eq := y = \frac{\ln(x)}{2} \tag{1.2}$$

$sol := solve(eq, x)$;
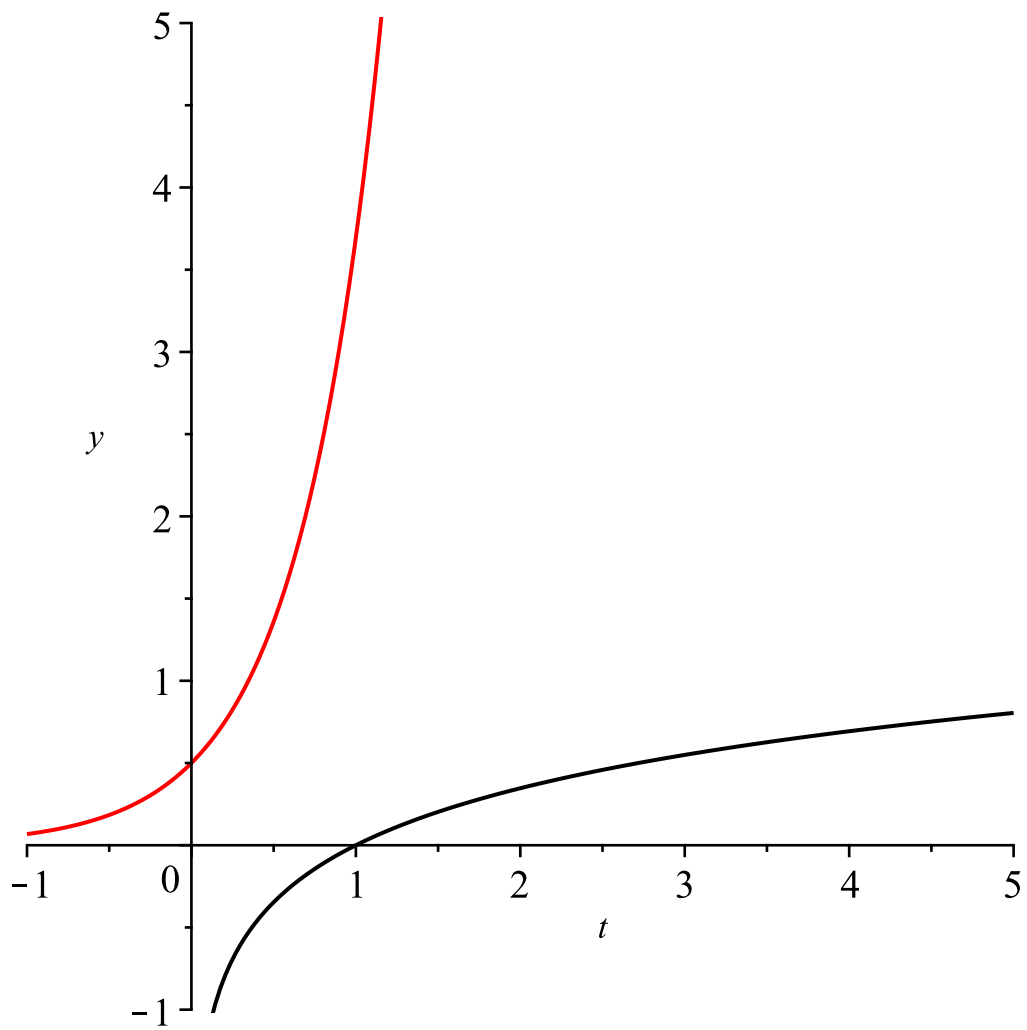
$$sol := e^{2y} \tag{1.3}$$

$f := unapply(rhs(eq), x)$;

$$f := x \mapsto \frac{\ln(x)}{2} \tag{1.4}$$

$g := unapply\left(\frac{sol}{2}, y\right)$;

$$g := y \mapsto \frac{e^{2y}}{2} \tag{1.5}$$

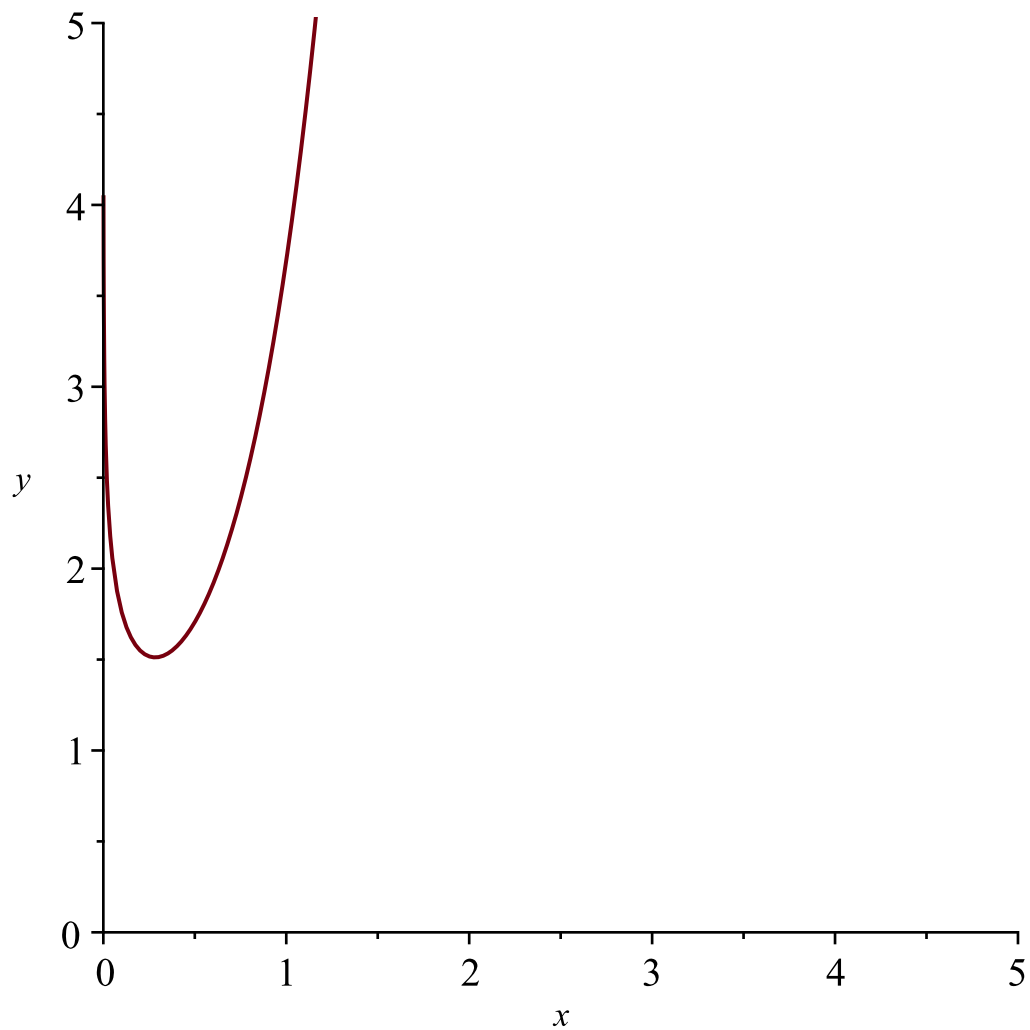$plot([f(t), g(t)], t = -1 .. 5, y = -1 .. 5, color = [black, red], scaling = constrained)$;

$h := x \rightarrow g(x) - f(x);$

$$h := x \mapsto g(x) - f(x) \tag{1.6}$$

$plot(h(x), x = 0 .. 5, y = 0 .. 5);$

$x0 := fsolve(h'(x) = 0);$

$$x0 := 0.2835716452 \tag{1.7}$$

$y0 := h(x0);$

$$y0 := 1.511756652 \tag{1.8}$$

▼ Exercise 2.

restart :
with(plots) :

$f := x \rightarrow -6\,x^2 + 12 \cdot x;$

$$f := x \mapsto -6\,x^2 + 12\,x \tag{2.1}$$

$g := x \rightarrow \dfrac{x^2 + 1}{x};$

$$g := x \mapsto \frac{x^2 + 1}{x} \tag{2.2}$$

$plot([f(x), g(x)], x = -10..10, y = -10..10, color = [black, red], scaling = constrained);$
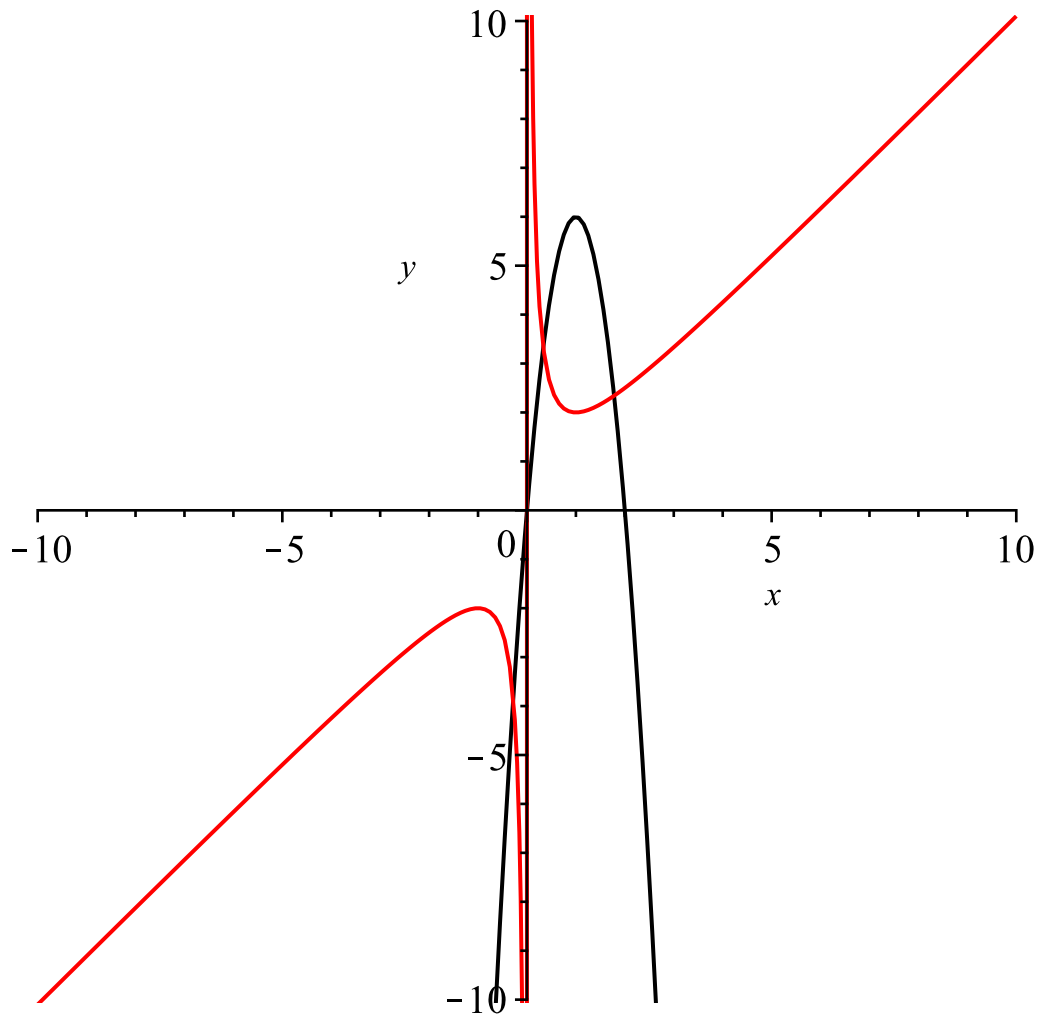
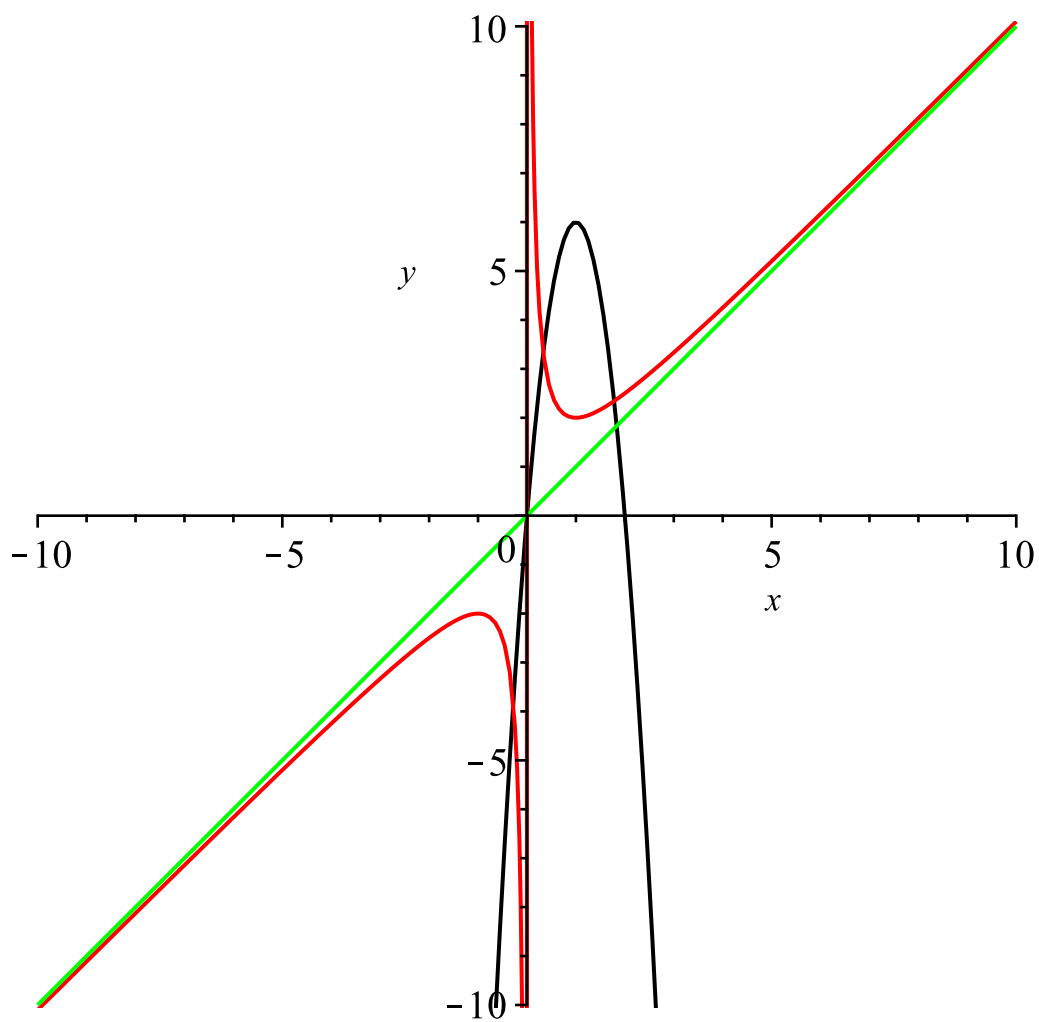$x0 := fsolve(f(x) - g(x), x = 0..1);$

$$x0 := 0.3333333333 \tag{2.3}$$

$y0 := f(x0);$

$$y0 := 3.333333333 \tag{2.4}$$

$x1 := fsolve(f(x) - g(x), x = 1..3);$

$$x1 := 1.780776406 \tag{2.5}$$

$y1 := f(x1);$

$$y1 := 2.34232922 \tag{2.6}$$

$$\int_{x0}^{x1} f(x) - g(x)\, dx;$$

$$3.934436339 \tag{2.7}$$

$plot([f(x), g(x), x], x = -10..10, y = -10..10, color = [black, red, green], scaling = constrained);$

$$\lim_{x \to \infty} (g(x) - x);$$

<div align="right">

(2.8)
</div>

0

$$\lim_{x \to -\infty} (g(x) - x);$$

<div align="right">

(2.9)
</div>

0

▼ Exercise 3.

restart :
with(*plots*) :
with(*DETools*);

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(3.1)
</div>

[*AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor,*
DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FunctionDecomposition,
GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM,
MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords,
PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols,

Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

$$deq := \frac{\mathrm{d}}{\mathrm{d}t}u(t) + \frac{u(t)}{2} = \mathrm{e}^{\frac{t}{3}};$$

$$deq := \frac{\mathrm{d}}{\mathrm{d}t}\,u(t) + \frac{u(t)}{2} = \mathrm{e}^{\frac{t}{3}} \tag{3.2}$$

DEplot(*deq*, *u*(*t*), *t* = 0 ..6, *u* = −2 ..4, *color* = *blue*, *arrows* = *medium*, *scaling* = *constrained*);
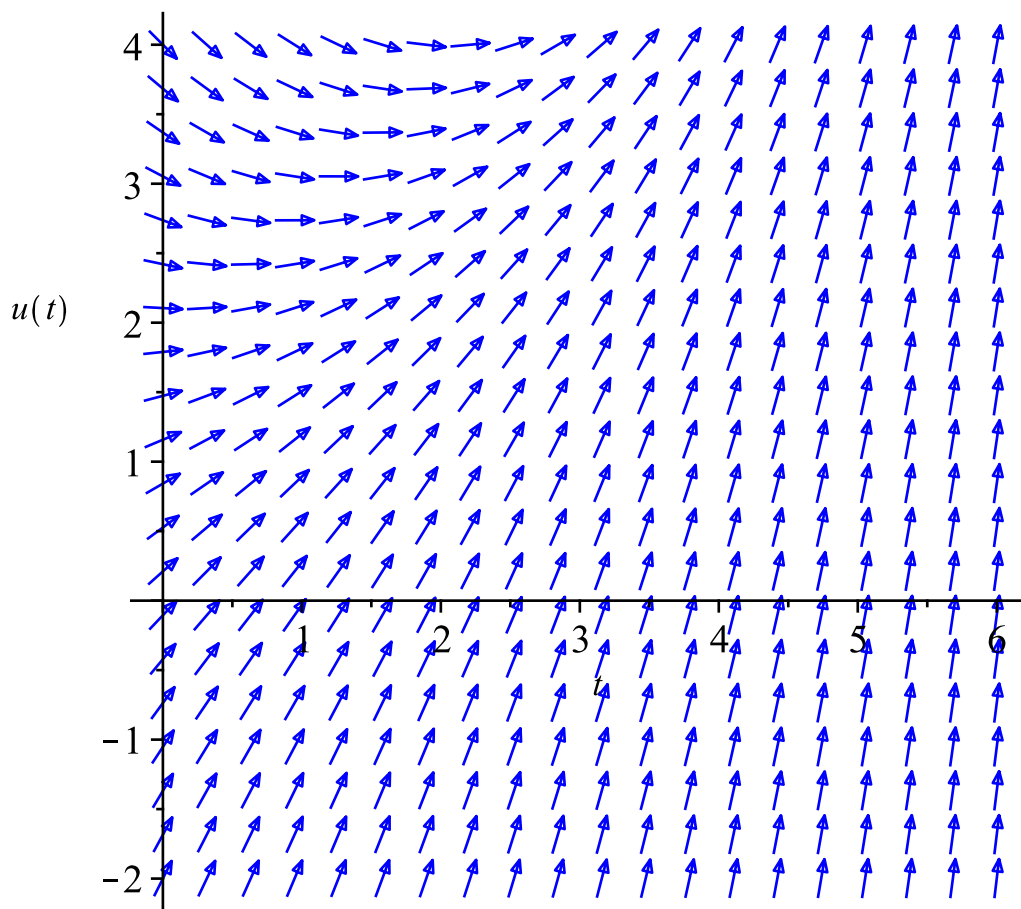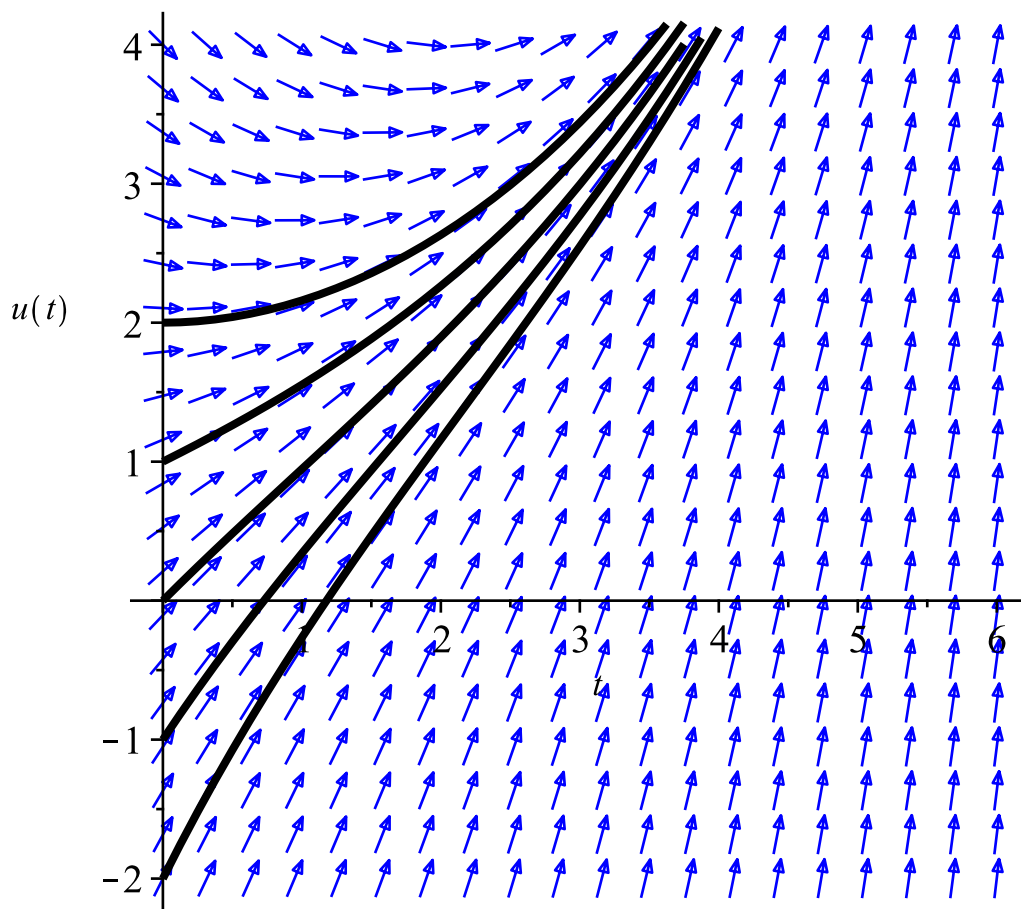
$inits := [seq(u(0) = k, k = -2 .. 2)];$

$$inits := [u(0) = -2, u(0) = -1, u(0) = 0, u(0) = 1, u(0) = 2]$$ **(3.3)**

$DEplot(deq, u(t), t = 0 .. 6, u = -2 .. 4, inits, color = blue, linecolor = black, arrows = medium, scaling = constrained);$

$sol := dsolve([deq, u(3) = 0], u(t))$

$$sol := u(t) = \left(\frac{6\,e^{\frac{5t}{6}}}{5} - \frac{6\,e^{\frac{5}{2}}}{5} \right) e^{-\frac{t}{2}} \qquad\qquad \textbf{(3.4)}$$

$v := unapply(rhs(sol), t);$

$$v := t \mapsto \left(\frac{6\,e^{\frac{5t}{6}}}{5} - \frac{6\,e^{\frac{5}{2}}}{5} \right) e^{-\frac{t}{2}} \qquad\qquad \textbf{(3.5)}$$

$v(4);$

$$\left(\frac{6\,e^{\frac{10}{3}}}{5} - \frac{6\,e^{\frac{5}{2}}}{5} \right) e^{-2} \qquad\qquad \textbf{(3.6)}$$

$\xrightarrow{\text{at 5 digits}}$

$$2.5740 \qquad\qquad \textbf{(3.7)}$$