

Exam Linear Algebra 1 part 1, AM1030
 Friday 18 september 2020, 15:00-15:50 uur.

EVERY ANSWER HAS TO BE MOTIVATED.

This exam is an ‘open book’ exam. That means you are allowed to use hardcopies of the reader, the book, handouts from brightspace and your own personal lecture notes. You are not allowed to use a computer, help from fellow students, help from other people and help from the internet. For every part of the exercise the maximum score is noted in the margin. The grade is determined by adding 20/9 points to your total score and dividing the result by 20/9.

- (0) 1. Om plechtig te beloven dat je bij het maken van dit open boek tentamen geen gebruik maakt van niet toegestane middelen en geen hulp inroept van derden dien je de volgende zin, inclusief het versienummer, op te schrijven en te ondertekenen:

Ik verklaar dat ik dit tentamen (versie 3241) zonder hulp van derden heb gemaakt, geheel conform het beleid van de TU Delft met betrekking tot plagiaat, bedrog en fraude.

- (7) 2. Given is a linear system with parameters α and β in $\mathbb{R}$ and augmented matrix

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & \alpha - 2 & 2 & \beta \\ 0 & 4 - 2\alpha & -\alpha & 2 \end{array} \right].$$

Find all values of α and β for which the system has no solutions. Give a clear explanation!

- (2) 3. Let

$$A^{-1} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$

If possible, find a matrix B such that

$$BA = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 0 & 2 \end{bmatrix}.$$

Explain your answer!

4. Given is the matrix A that we already row reduced to an echelon form U :

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 2 \\ 3 & 2 & 4 & 1 & 0 \\ -1 & 3 & -5 & -1 & 3 \\ 2 & -1 & 5 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 3 & 2 \\ 0 & 1 & -1 & 4 & 5 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = U$$

The columns of A are denoted by $\mathbf{a}_1, \dots, \mathbf{a}_5$, so for example $\mathbf{a}_2 = (0, 2, 3, -1)^T$.

- (3) (a) Find the dimension of $W = \text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$. Don't forget the explanation!
 (4) (b) Is $\{\mathbf{a}_2, \mathbf{a}_4, \mathbf{a}_5\}$ a basis for W ? Why or why not?

- (4) 5. Let A and B be symmetric $n \times n$ matrices such that $AB = BA$. Prove that BA is symmetric.