Technische Universiteit Delft
Faculteit Elektrotechniek, Wiskunde en Informatica
Mekelweg 4, Delft

Exam Kaleidoscope, (TW1021 and TW1021A)
5 October, 2 November 2018, for two hours.

The use of a mobile phone or a calculator is forbidden.

Every answer has to be motivated.

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1. Calculate the solutions of the following equations, write them in the form $z = a + bi$ with a, $b \in \mathbb{R}$, and draw them in the complex plane.

(5) (a) $z^3 = 8i$,

(4) (b) $\mathrm{e}^{iz} = 8i$,

(4) (c) $z^2 + (2 + 2i)z + 1 + 2i = 0$.

2. Given is the curve $\mathcal{C}$ with equation

$$|z + 2i| = \sqrt{2}|z + 1|. \tag{1}$$

(3) (a) Let $z = x + iy$ with x, $y \in \mathbb{R}$ and write (2) as an equation in x and y.

(1) (b) Draw $\mathcal{C}$ in the complex plane.

3. Consider the initial value problem

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = u(u^2 - 1) & \text{(2a)} \\ u(0) = \alpha & \text{(2b)} \end{cases}$$

(2) (a) Determine the equilibrium solutions of the differential equation (2a).

(3) (b) Sketch the equilibrium solutions of (2a) and the solutions of the initial value problem (2a),(2b) for $\alpha = -2$, $\alpha = -\frac{1}{2}$, $\alpha = \frac{1}{2}$ and $\alpha = 2$ in one picture.

(2) (c) Which of the equilibrium solutions are stable and which are unstable?

4. Consider the initial value problem

$$\begin{cases} ty' + 2y = \dfrac{\sin t}{t} \qquad (t > 0) & \text{(3a)} \\ y(\dfrac{\pi}{2}) = 1 & \text{(3b)} \end{cases}$$

(4) (a) Solve the homogeneous equation corresponding to (3a).

(4) (b) Determine the general solution of (3a) by using 'variation of parameters'.

(2) (c) Determine the solution of the initial value problem (3a), (3b).

5. Consider the differential equation

$$u'' - 4u' + 4u = 5\sin t \tag{4}$$

(4) (a) Prove that the functions u_1 and u_2 given by $u_1(t) = \mathrm{e}^{2t}$ and $u_2(t) = t\mathrm{e}^{2t}$ are solutions of the homogeneous differential equation corresponding to (4).

(1) (b) Determine the general solution of the homogeneous differential equation corresponding to (4). It can easily be shown that there are constants A and B such that y_p, with $y_p(t) = A\cos(t) + B\sin(t)$, is a particular solution of (4). [1]

(5) (c) Determine A and B and give the general solution of (4).

Mark: (total number of points + 4)/4 rounded up to the nearest half.

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[1] This is known as 'the method of undetermined coefficients'.