Test 1 Mathematical Structures AM1010
Wednesday October 2, 2019, 9:00-10:00

TU Delft

No calculators allowed. Write the solutions in the fields provided. The grade is (score+4)/4.

Exercise [ ] continued (extra space)

1a Write down the truth table for the expression $(q \Rightarrow p) \wedge (\sim p)$. 3

1b Give a statement R in terms of p and q, expressed without using $\Rightarrow$ and $\wedge$ such that $(q \Rightarrow p) \wedge (\sim p) \Leftrightarrow R$ is a tautology. For example R could be $(\sim q) \vee p$. 2

You don't have to explain your answer.

2a Give the definition of a partition of a set. 2

A partition of the set S is ...

2b Give an example of a partition of the set $S = \{5, 8, 12\}$. 1

3 Let A, B, and C be sets. Show that $(A \setminus C) \cup (B \setminus C) \subseteq (A \cup B) \setminus C$. 6

4 We define a relation R on $\mathbb{N}$ as nRm iff there are odd integers p and q such that $\frac{n}{m} = \frac{p}{q}$. You may assume that this relation is transitive. Remember to give a proof for all your answers.

4a Is the relation R reflexive? 2

4b Is the relation R symmetric? 3

4c Is the relation R an equivalence relation? If so, give a simple expression for the equivalence class E_1. 2

5a Write down the negation of 4

$$\forall x \in \mathbb{R}\, \exists y \in \mathbb{R}\, \forall z \in \mathbb{R} : z \geq y \Rightarrow x \leq 2z$$

in simplified form (the negation symbol itself is not allowed in your answer).
You only have to give your anwer, no explanation required.

5b Prove or disprove the statement from 5a. 3

6a Show or disprove: $f : \mathbb{R} \to \mathbb{R}$ is increasing implies it is injective. 4

Note: f is increasing if $\forall x, y \in \mathbb{R} : x > y \Rightarrow f(x) > f(y)$.

6b Show or disprove: If $f : \mathbb{R} \to \mathbb{R}$ is injective, then it is either increasing or decreasing. 4

Examiner resposible: Fokko van de Bult
Examination reviewer: Wolter Groenevelt