Midterm Mathematical Structures AM1010
Monday November 4, 2019, 9:00-11:00

No calculators allowed. Write the solutions in the fields provided. The grade is (score+6)/6.

Exercise [ ] continued (extra space)

Exercise 1 is at the bottom of this page!

Exercise [ ] continued (extra space)

1. (a) Write the statement below using quantifiers, logical operators, equations, and expressions such as $a \in \mathbb{R}$. 4

There exists an integer x such that xy is an integer multiple of 7 for all rational numbers y.

Don't use words or the divisor symbol $n \mid m$. No explanation necessary.

(b) Write down the negation of the above statement (again using quantifiers, logical operators, equations, and expressions such as $a \in \mathbb{R}$). No explanation necessary. 3

2. Let $f : A \to B$ be a function. Suppose $C \subseteq A$ and $D \subseteq B$. Show that 6

$$f(C) \setminus D = f(C \setminus f^{-1}(D))$$

using the definitions of image and pre-image.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Define a relation R on $\mathbb{R}$ by saying xRy holds whenever $xf(y) = yf(x)$.

(a) Is this relation reflexive (for any function f)? Prove or disprove. 2

(b) Is this relation symmetric (for any function f)? Prove or disprove. 2

(c) Is this relation transitive (for any function f)? Prove or disprove. 3

(d) Give an example of a function f for which this relation is an equivalence relation and show that this is the case. 2

4. Suppose $f : [0, 1] \to [0, 1]$ is increasing and $f(0) = 0,\ f(1) = 1$. Prove or disprove: f is surjective. 4

   Note: f is increasing if $\forall x, y \in \mathbb{R} : x > y \Rightarrow f(x) > f(y)$.

5. In this exercise you only have to provide the final answer, no explanation necessary.

   (a) Write $\bigcap_{n\in\mathbb{N}} (2 - \frac{1}{n}, 4 - \frac{1}{n})$ as a finite union of intervals and finite sets. 2

   (b) Write $\bigcup_{n\in\mathbb{N}} (2 - \frac{1}{n}, 4 - \frac{1}{n})$ as a finite union of intervals and finite sets. 2

6. Prove using induction that $2n^3 - 3n^2 + n$ is a multiple of 6 for all $n \in \mathbb{N}$. 8

7. Complete the definition of $\lim s_n = s$ for a real number s. 2

The sequence (s_n) converges to s when ...

8. Prove using the definition of infinite limits that $\lim \frac{n^2+5}{n+3} = \infty$. 8

9. Suppose (s_n) is a convergent sequence and (t_n) is divergent. Prove or disprove the following statements

   (a) $(s_n t_n)$ is divergent. 3

   (b) $(s_n + t_n)$ is divergent. 3

Examiner resposible: Fokko van de Bult
Examination reviewer: Wolter Groenevelt and Anna Geyer