Test 2 Mathematical Structures AM1010
Friday December 13, 2019, 9:00-10:00

No calculators allowed. Write the solutions in the fields provided. The grade is (score+4)/4.

Exercise [ ] continued (extra space)

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1 Let $S = \{\frac{n+2}{n} \; : \; n \in \mathbb{N}\}$. Determine $\inf(S)$ and $\sup(S)$. Prove your answer in detail. 5

$\inf(S) =$

Explanation for inf:

$\sup(S) =$

Explanation for sup:

2 Consider = the sequence defined recursively as $a_{n+1} = \sqrt{3a_n + 1}$ for $n \geq 1$, and $a_1 = 1$. You may assume that the sequence is monotone.

2a Show that the sequence (a_n) converges. (If you use induction you need to write down a proper induction proof.) 9

2b Determine the limit $a = \lim a_n$ (for this part you can assume (a_n) converges). 4

3 Formulate the completeness axiom for the real numbers. 2

(Give the one from this course, not the one from AM2090: Real analysis.)

4 What is the name of the axiom which says that $\forall x, y \in \mathbb{R} : x + y = y + x$? 2

5 Show that the sets $S = (0, \infty)$ are $T = [0, \infty)$ equinumerous by giving an explicit bijection. You don't have to show it is truly a bijection. 4

6 Let (s_n) be a bounded sequence: That is, there exists M such that $|s_n| < M$ for all n. Moreover write $\limsup s_n = s$.

6a Give an example of a bounded sequence (s_n) such that $\limsup(s_n^2) \neq s^2$. Show that this is the case by giving s_n, s and $\limsup(s_n^2)$. You don't have to give a proof that the lim sup's are what you say they are. 4

$s_n =$

$s = \limsup s_n =$

$\limsup(s_n^2) =$

6b Suppose $s_n > 0$. Show that now $\limsup(s_n^2) = s^2$. 6

Examiner responsible: Fokko van de Bult
Examination reviewer: Wolter Groenevelt, Rik Versendaal, Anna Geyer.