Technische Universiteit Delft
Fac. Elektrotechniek, Wiskunde en Informatica

Exam for Valuation of Derivatives, Wi 3405TU

Friday April 15th 2016, 13:30 - 15:30 **(2 hours examination)**

1. The Black-Scholes formula for the value of a put option is given by:

$$\begin{aligned} P(S,t) &= S(N(d_1)-1) + Ee^{-r(T-t)}(1-N(d_2)), \text{ with} \\ d_{1,2} &= \frac{\log(S/E) + (r \pm \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \end{aligned}$$

(plus-sign for d_1, minus-sign for d_2)

a. Explain all parameters and variables in this formula, and give the market assumptions under which this option formula is derived.

b. Derive

$$\frac{\partial P(S,t)}{\partial S},$$

and determine the limiting behaviour for $t \to T^-$. You may use the identity:

$$SN'(d_1) - e^{-r(T-t)}EN'(d_2) = 0,$$

in which $N'(\cdot)$ represents the derivative of $N(\cdot)$.

c. Give financial arguments that explain the value of $\partial P(S,t)/\partial S$ at expiry for an in-the-money option, as well as $\partial P(S,t)/\partial S$ for an out-of-the-money option.

2. An *up-and-out barrier call* option is a barrier option which pays out nothing, when the option is knocked out; Otherwise the payoff is paid at time $t = T$. Assume that the asset follows the process that leads to the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0.$$

a. Give the appropriate computational domain, the corresponding boundary conditions and the payoff function for the up-and-out barrier call option. Transform the equations by $\tau = T - t$ into forward equations in time.

b. Give a put-call parity relation for this option, for $t_0 \le t \le T$.

c. Write down the implicit "backward difference in time, central difference in space" (BTCS) discretization of the up-and-out barrier call option, as a difference equation.

3. Consider the following piece of Matlab code:

```
randn('state',100)
clf
S = 1; mu = 0.05; si = 0.5; L = 1e2; T = 1;
dt = T/L; M = 50;
tvals = [0:dt:T];
Sv = S*cumprod(exp((mu-0.5*si*si)*dt+...
si*dt*randn(M,L)),2);
Sv = [S*ones(M,1) Sv];
plot(tvals,Sv)
title('???')
xlabel('???'), ylabel('???')
```

a. The mathematics in the code above is not correct. Detect the mathematical error, give the result of this error and how should it be improved? Fill in the missing labels for the corrected code, indicated by the question marks.

Check whether your name and study number are on each of the pages with solutions!