Technische Universiteit Delft
Fac. Elektrotechniek, Wiskunde en Informatica

Exam for Valuation of Derivatives, Wi 3405TU

Thursday April 18th 2019, 18:30 - 20:30 **(2 hours examination)**

1. An option strategy, which is called the bull spread, is to hold a call option with exercise price E_1 and, for the same asset and expiry date, to write a call option with exercise price E_2, where $E_2 > E_1$.

a. Determine the pay-off function at the expiry date of this strategy and draw the corresponding payoff diagram.

Under which asset movements will the holder of such a spread benefit from this position?

b. Replicate the bull spread construction with the help of put options.

2. The Black-Scholes formula for the value of a put option is given by:

$$\begin{aligned} P(S,t) &= S(N(d_1)-1) + Ee^{-r(T-t)}(1-N(d_2)), \text{ with} \\ d_{1,2} &= \frac{\log(S/E) + (r \pm \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \end{aligned}$$

(plus-sign for d_1, minus-sign for d_2)

a. Derive

$$\frac{\partial P(S,t)}{\partial S},$$

and determine the limiting behaviour for $t \to T^-$. You may use the identity:

$$SN'(d_1) - e^{-r(T-t)}EN'(d_2) = 0,$$

in which $N'(\cdot)$ represents the derivative of $N(\cdot)$.

b. Give financial arguments that explain the value of $\frac{\partial P(S,t)}{\partial S}$ at time T for an in-the-money option, as well as $\frac{\partial P(S,t)}{\partial S}$ for an out-of-the-money option.

c. Give the definition of implied volatility. Explain how we can compute the implied volatility of a call option.

d. Consider the put option to be a function of volatility, i.e., $P = P(\sigma)$. Compute

$$\lim_{\sigma\to\infty} P(\sigma), \quad \lim_{\sigma\to 0^+} P(\sigma).$$

3. Consider the following piece of Matlab code:

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 sig=0.5; r=0.1; E=1; S0=1; T=0.5; L=1e2; M=50; dt=T/L;
W = randn(M,L);
Svals = S0*cumprod(((r-sig^2/2)*dt+sig*sqrt(dt)*W),2);
Svals = [S0*ones(M,1) Svals];
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a. Describe what the code aims to compute, and indicate where a mathematical error is being made, which leads to an incorrect answer (and how to improve it?).

b. Describe three ways to determine the correctness and accuracy of the statistical answers, as obtained from such a code.

Check whether your name and study number are on each of the pages with solutions!