Technische Universiteit Delft
Fac. Elektrotechniek, Wiskunde en Informatica

Examination Option Valuation Methods, Wi 3405TU

Friday January 26nd 2018, 9:00 - 11:00 **(2 hours examination)**

1. a. Describe an Asian fixed strike call option, with a weekly updated arithmetic average, in terms of contract details. For which financial reason would a writer want to sell such an Asian option; and for which reason would an investor wish to buy it?

b. Let $C_1^{fix}(S,t)$ and $P_1^{fix}(S,t)$ denote prices of an Asian fixed strike call and put, respectively, with strike price E. Let $C_2^{float}(S,t)$ and $P_2^{float}(S,t)$ denote prices of a floating strike Asian call and put, and let $C_3(S,t)$ and $P_3(S,t)$ denote prices of a European call and put option, with strike price E. All options have expiry date T. Show that

$$C_1^{fix}(S,t)+C_2^{float}(S,t)-C_3(S,t) = P_1^{fix}(S,t)+P_2^{float}(S,t)-P_3(S,t).$$

2. Consider the following partial differential equation, for unknown $u(x,\tau)$,

$$\begin{aligned} \frac{\partial u}{\partial \tau} &= \frac{\partial^2 u}{\partial x^2},\ 0 \le x \le L,\ 0 \le \tau \le T. \\ u(x,0) &= b(x), \\ u(0,\tau) &= 0,\ u(L,\tau) = c(\tau), \end{aligned} \tag{1}$$

with $b(x)$ and $c(\tau)$ pre-specified functions. We discretize this equation on a computational grid with N_x points in x-direction and N_τ points in τ-direction, to find $U_j^i \approx u(jh, ik)$ by the Crank-Nicolson discretization, with time index $0 \le i \le N_\tau$ and spatial index $0 \le j \le N_x$, giving

$$2(1+\nu)U_j^{i+1} = \nu U_{j+1}^{i+1} + \nu U_{j-1}^{i+1} + \nu U_{j+1}^i + 2(1-\nu)U_j^i + \nu U_{j-1}^i \tag{2}$$

with $\nu = k/h^2$, and the mesh sizes $h = 1/N_x$ and $k = 1/N_\tau$.

a. Write down the Crank-Nicolson discretization as a matrix equation, $\hat{B}\mathbf{U}^{i+1} = \hat{F}\mathbf{U}^i + \mathbf{r}^i$, for $0 \le i \le N_t - 1$, by giving the details of the matrices $\hat{B}, \hat{F}$ and of vector $\mathbf{r}^i$.

b. When we deal with the Black-Scholes equation with $r = 0$ and $\tau = T - t$, i.e.,

$$\frac{\partial u}{\partial \tau} = \frac{1}{2} S^2 \sigma^2 \frac{\partial^2 u}{\partial S^2},$$

but with the same boundary and initial condition as in (1), how would the matrices and vector that were obtained under **2**a. change?

Z.O.Z.

c. Apply the von Neumann stability analysis to equation (2), based on $U_j^i = \xi^i e^{\mathbf{i}\beta jh}$ (with $\mathbf{i}$ the unit imaginary number). Show that the method is unconditionally stable.

3. The following Matlab code is given:

```
%%%%%%%%%% Problem and method parameters %%%%%%%%%%%%
S = 3; E = 2; T = 1; r = 0.05; sigma = 0.3;
M = 400; dt = T/M; p =0.5;
u = exp(sigma*sqrt(dt) + (r-0.5*sigma^2)*dt);
d = exp(-sigma*sqrt(dt) + (r-0.5*sigma^2)*dt);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

W = max(E-S*d.^([M:-1:0]').*u.^([0:M]'),0);

for i = M:-1:1
    W = exp(-r*dt)*(p*W(2:i+1) + (1-p)*W(1:i));
end

disp('Option value is'), disp(W)
```

a. What is the computational technique in this Matlab code and what is computed? Explain the parameters u, d, p in the method.

b. Adapt the Matlab code to value an American put option, and explain, in pseudo-code, how the coordinates of the early-exercise boundary can also be determined as output.

c. Describe how to value a down-and-out barrier put option, with barrier level $B = 1$, by the method from the above Matlab code, and adapt the code accordingly.

Place your name and study number on each page with solutions.