Technische Universiteit Delft
Fac. Elektrotechniek, Wiskunde en Informatica

Examination Valuation of Derivatives, Wi 3405TU

Friday January 27nd 2017, 9:00 - 11:00 **(2 hours examination)**

1. Consider the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0,$$

with a payoff function as the final condition at $t = T$.

a. Show that the transformations $S = e^y$, $\tau = T - t$, and $u(S,t) = e^{r\tau}V(X,t)$, $y = x + (r - \frac{1}{2}\sigma^2)\tau$ result in the following heat equation for unknown $u(\tau, x)$,

$$\frac{\partial u}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2},$$

b. Write down the explicit finite difference scheme, FTCS, for the heat equation. Show that the discrete scheme is second order accurate in space and first order accurate in time.

c. Show that the von Neumann stability condition takes the form $\sigma^2 k \le h^2$.

2. A double barrier option has two barriers, one above and one below the current stock price. In a *one-touch double barrier put option*, the option is knocked out, resulting in a valueless option, if at least one of the barriers is breached during the life of the option.

We assume that the asset follows the process that leads to the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0.$$

a. Why would an investor buy such an option, and why would a writer sell such an option?

b. Give the appropriate computational domain, the corresponding boundary conditions and the payoff function for a one-touch double barrier put option. Transform the equations by $\tau = T - t$ into forward equations in time.

c. We define a vector of option values for time t_i as $\mathbf{V}^i = [V_1^i, V_2^i, \ldots, V_{N_x-1}^i]^T$. Write down the matrices B and F and vector $\mathbf{q}$ of the matrix-vector representation $B\mathbf{V}^{i+1} = F\mathbf{V}^i + \mathbf{q}^{i+1}$ of BTCS, for this barrier option.

d. Why does it not make much sense to have $B < E$ in an up-and-in call option?

Z.O.Z.

3. The following Matlab code is given:

```
%%%%%%%%%% Problem and method parameters %%%%%%%%%%%%%
S = 3; E = 2; T = 1; r = 0.05; sigma = 0.3;
M = 400; dt = T/M; p =0.5;
u = exp(sigma*sqrt(dt) + (r-0.5*sigma^2)*dt);
d = exp(-sigma*sqrt(dt) + (r-0.5*sigma^2)*dt);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Time T option values
W = max(E-S*d.^([M:-1:0]').*u.^([0:M]'),0);

% Work back to option value at time zero
for i = M:-1:1
    W = exp(-r*dt)*(p*W(2:i+1) + (1-p)*W(1:i));
end

disp('Option value is'), disp(W)
```

– What is the computational technique in this Matlab code and what is computed? Adapt the Matlab code above to value a Bermudan put option with four exercise dates, at $t = 0.25$, $t = 0.5$, $t = 0.75$ and $T = 1$.

Place your name and study number on each page with solutions.