Exam Relaxations and Heuristics (WI4515)

22 January 2019, 13.30–16.30 (3 hours).

The exam consists of 5 questions worth 10 points each. Your grade is given by $1 + \frac{9p}{50}$, where p is the total number of points obtained.
Devices, notes, books etc. are **not** permitted.
The total number of pages of this exam is 2. **Good luck!**

[10pts] 1. A set of n jobs must be carried out on a single machine that can do only one job at a time. Each job j takes p_j hours to complete. Given job weights w_j for $j = 1, \ldots, n$, in what order should the jobs be carried out so as to minimize the weighted sum of their start times? Formulate this scheduling problem as a mixed integer program and argue that your formulation is correct.

2. For each set X below and for each point x, find a valid inequality for X cutting off point x.

[2pts] (a) $X = \{ (x_1, x_2, y) \in \mathbb{R}^2_+ \times \{0,1\} : x_1 + x_2 \le 2y,\ x_1, x_2 \le 1 \}$, $(x_1, x_2, y) = (1, 0, 0.5)$.

[2pts] (b) $X = \{ (x, y) \in \mathbb{R}_+ \times \mathbb{Z}_+ : x \le 9,\ x \le 4y \}$, $(x, y) = (9, 9/4)$.

[2pts] (c) $X = \{ x \in \{0,1\}^5 : 9x_1 + 8x_2 + 6x_3 + 6x_4 + 5x_5 \le 14 \}$, $x = (0, 5/8, 3/4, 3/4, 0)$.

[2pts] (d) $X = \{ x \in \{0,1\}^5 : 7x_1 + 6x_2 + 6x_3 + 4x_4 + 3x_5 \le 14 \}$, $x = (1/7, 1, 1/2, 1/4, 1)$.

[2pts] (e) $X = \{ x \in \{0,1\}^5 : 12x_1 - 9x_2 + 8x_3 + 6x_4 - 3x_5 \le 2 \}$, $x = (0, 0, 1/2, 1/6, 1)$.

3. A *stable set* in a graph $G = (V, E)$ is a subset of V of which no two vertices are adjacent. A *clique* in G is a subset of V in which any two vertices are adjacent. The *stable set polytope* of G is the convex hull of the incidence vectors of stable sets in G.

[2pts] (a) What is the dimension of the stable set polytope?

[3pts] (b) Let $C \subseteq V$ be a clique in G. Show that the clique inequality

$$\sum_{v \in V} x_v \le 1$$

is valid for the stable set polytope of G.

[5pts] (c) Show that the clique inequality above induces a facet of the polytope if and only if C is an inclusionwise-maximal clique. (An *inclusionwise-maximal clique* is a clique that is not a proper subset of any other clique.)

CONTINUE ON THE OTHER SIDE...

[10pts] 4. Given a graph $G = (V, E)$, a depot node 0, edge costs c_e for each $e \in E$, K identical vehicles of capacity C, and client orders d_i for $i \in V \setminus \{0\}$, we wish to find a set of subtours (cycles) for each vehicle such that

(i) each subtour contains the depot,

(ii) together the subtours contain all the nodes,

(iii) the subtours are disjoint on the node set $V \setminus \{0\}$,

(iv) the total demand on each subtour (the total amount delivered by each vehicle) does not exceed C.

Formulate the integer programming master and subproblems for this problem.

[10pts] 5. Define the set-covering problem and describe a greedy heuristic for it.

End of the exam